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MICROMECHANICAL STUDY OF DAMAGE GROWTH AND PERMEABILITY VARIATION IN BRITTLE ROCKS

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Abstract: This paper presents a coupled model for anisotropic damage and permeability evolution by using a micro-macro approach. The damage state is represented by a space distribution of microcrack density. The evolution of damage is directly linked to the propagation condition of microcracks. The macroscopic free enthalpy function of cracked material is obtained by using micromechanical considerations. It is assumed that the microcracks exhibit normal aperture which is associated with the crack growth due to the asperity of crack faces. By using Darcy's law for macroscopic fluid flow and assuming laminar flow in microcracks, the overall permeability of the representative volume element is obtained by an averaging procedure taking into account the contribution of crack aperture in each orientation.

Streszczenie: Przedstawiono sprzężony model anizotropowego uszkodzenia i zmiany przepuszczalności, stosując podejście mikro-makro. Stan uszkodzenia jest odwzorowany przez rozkład przestrzenny gęstości mikropęknięcia. Ewolucja uszkodzenia wiąże się bezpośrednio z warunkiem rozprzestrzeniania się gęstości mikrouszkodzeń. Makroskopową funkcję entalpii swobodnej spękanego materiału otrzymuje się na podstawie rozważań mikromechanicznych. Zakłada się, że mikrouszkodzenia mają zwykłą szczelinę, co jest związane z powiększaniem się pęknięcia wskutek chropowatości powierzchni czołowej pęknięć. Używając prawa Darcy'ego do opisu makroskopowego przepływu cieczy i zakładając, że jest on laminarny w mikrouszkodzeniach, otrzymano całkowitą przepuszczalność reprezentatywnego elementu trójwymiarowego za pomocą uśredniającej procedury, w której uwzględnia się udział szczeliny pęknięcia w każdym kierunku.

Резюме: Применяя подход микро-макро, представили сопряженную модель анизотропного повреждения и изменения проницаемой способности. Состояние повреждения отражено путем пространственного распределения плотности микротрещин. Эволюция повреждения непосредственно связана с условием распространения плотности микроповреждений. Макроскопная функция свободной энтальпии трещиноватого материала получена на основе микромеханических рассуждений. Предполагается, что микроповреждения обадают обычной трещинойб что связано с увеличением трещины из-за шероховатости лобовой поверхности трещин. Используя закон Дарси для описания макроскопного протекания жидкости и предполагая, что оно ламинарно в месте микроповреждений, получили полную проницаемую способность представительного трехразмерного элемента с помощью усредняющей процедуры, в которой учитывается участие щели трещины в каждом направлении.

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1. INTRODUCTION

In brittle geomaterials like rocks and concrete, damage by nucleation and growth of microcracks is an essential mechanism of deformation and failure. The induced damage not only affects the mechanical properties of materials, but also the flow and conductivity properties. The variation of permeability with the growth of microcracks is one of the most significant phenomena to be taken into account in many engineering applications, for example, the storage of nuclear wastes, the stability of rock slopes and hydraulic dams, and the long-term durability of concrete structures.

A number of constitutive models have been proposed for the description of induced damage in geomaterials (we do not give an exhaustive list of these models here). They can be roughly separated into two classes: phenomenological models and micromechanical models.

Generally, the advantage of the micromechanical models lies in the possibility of accounting for physical mechanisms involved in material damage. However, the numerical implementation of these models in view of engineering application is not easy and the associated computation procedure is usually time-consuming. On the other hand, the phenomenological models provide us with simple and unified mathematical formulations. These models can be easily implemented in a computer code and then used as a powerful tool for engineering analyses. However, some assumptions and concepts used in the phenomenological models, for instance, the effective stress concept, are not clearly based on physical backgrounds. On the other hand, various approaches have been proposed for the estimation of permeability in fractured media, involving empirical and statistical investigations (ODA [9], LEE et al. [7], ZHANG et al. [19], SUZUKI et al. [15], CHEN et al. [3], DOOLIN and MAULDON [4], SCHULZE et al. [13], ODA et al. [10], BERKOWITZ [2], WANG and PARK [16]). However, these investigations are generally performed for a given distribution of fractures in rocks, and they are not coupled with the evolution of mechanical behaviours of materials. There are very few studies on the coupled modelling of mechanical damage and permeability evolution.

In the first part of this paper, we propose to develop a new anisotropic damage model for brittle rocks essentially subjected to compressive stresses. The model proposed will be based on the relevant micromechanics analyses in order to take into account main physical mechanisms involved in the microstructure scale. In the second part, the mechanical model is extended to the description of variation of permeability due to the growth of microcracks. The variation of permeability is then directly associated with the mechanical damage of material. Throughout the paper, the following notations for tensorial calculations will be used: $(\vec{a} \otimes \vec{b})_{ij} = a_i b_j$, $(\underline{\underline{A}}.\vec{b})_i = A_{ij} b_j$, $(\underline{\underline{A}} : \underline{\underline{B}}) = A_{ij} B_{ji}$, $(\vec{a}.\vec{b}) = a_i b_i$.

2. FORMULATION OF THE DAMAGE MODEL

In this paper, it is assumed that rocks are subjected to compression-dominated stresses. The crack density remains small and the interaction between microcracks can be neglected before the onset of coalescence of microcracks. The initial behaviour of materials is isotropic and the anisotropy is fully induced by preferential distribution of microcracks.

2.1. FREE ENTHALPY AND CONSTITUTIVE EQUATIONS

In brittle materials like rocks, damage by nucleation and growth of microcracks is the essential dissipation mechanism. Plastic deformation due to dislocation-like sliding can be neglected. Macroscopic irreversible strains are developed due to residual opening and mismatch of microcracks during loading–unloading process. Let us consider now a representative volume element (RVE) of the cracked material. The letter Ω stands for the volume of RVE. The RVE is composed of an elastic solid matrix, which is weakened by a number of sets of microcracks appearing in different directions. The RVE is subjected to a uniform stress field $\boldsymbol{\sigma}$ on its boundary. For the simplicity of mathematical formulation, we first consider a single family of N similar microcracks inside the RVE, oriented in the direction defined by the unit normal vector $\vec{n}$. In the same family, all the cracks have the same geometrical form. The vector of displacement jump on each microcrack is defined as follows:

$$\vec{b} = \vec{u}^{+} - \vec{u}^{-}, \tag{1}$$

where $\vec{u}^{+}$ and $\vec{u}^{-}$ are respectively the displacement vectors on the two opposite faces of the crack. The macroscopic strain tensor of the REV can be determined by:

$$\boldsymbol{\varepsilon} = \mathbf{S}^0 : \boldsymbol{\sigma} + \frac{1}{2}\frac{N}{\Omega}\int_{\Gamma^+} (\vec{b} \otimes \vec{n} + \vec{n} \otimes \vec{b})ds, \tag{2}$$

where Γ^+ denotes the crack face with the positive normal unit vector $\vec{n}$. $\mathbf{S}^0$ is the initial elastic compliance tensor of undamaged material. In the case of penny-shaped cracks, the unit normal $\vec{n}$ is constant along the crack surface and the vector $\vec{b}$ is taken as the average displacement jump over the crack. The elastic free enthalpy can be expressed by:

$$w_c = \frac{1}{2}\boldsymbol{\sigma} : \mathbf{S}^0 : \boldsymbol{\sigma} + \frac{N}{\Omega}(\boldsymbol{\sigma}.\vec{n}).\vec{b}(\pi r^2). \tag{3}$$

The variable r denotes the radius of microcracks. According to the fundamental work by KACHANOV [6], the displacement jump can be decomposed into a normal component and a shear component. The two components can be related respectively to the normal stress and shear stress vectors applied to the crack:

$$\vec{b} = \beta(\vec{n}.\sigma.\vec{n})\vec{n} + \gamma\left[\sigma.\vec{n} - (\vec{n}.\sigma.\vec{n})\vec{n}\right]. \qquad (4)$$

The two coefficients involved in (4) are given by KACHANOV [6]:

$$\beta = \frac{16(1-\nu_0^2)}{3E_0}\frac{r}{\pi}, \quad \gamma = \frac{2}{2-\nu_0}\beta . \qquad (5)$$

E_0 and ν_0 are respectively the initial Young's modulus and the Poisson ratio of undamaged material. According to (4), the normal displacement jump is proportional to the normal stress applied to the crack. As the normal displacement jump must be positive (opened cracks) or zero (closed cracks), the following closure conditions of crack have to be prescribed:

$$\begin{cases} (\vec{n}.\sigma.\vec{n}) > 0, & \text{opened cracks,} \\ (\vec{n}.\sigma.\vec{n}) \le 0, & \text{closed cracks.} \end{cases} \qquad (6)$$

By introducing this closure condition and relation (4) into (3), the free enthalpy function becomes:

$$w_c = \frac{1}{2}\sigma : \mathbf{S}^0 : \sigma + \omega h\left\{\left(1-\frac{\nu_0}{2}\right)\langle \vec{n}.\sigma.\vec{n}\rangle^+ \vec{n} + [(\sigma.\vec{n}) - (\vec{n}.\sigma.\vec{n})\vec{n}]\right\}.(\sigma.\vec{n}). \qquad (7)$$

The bracket $\langle x\rangle^+$ defines the positive cone of the normal stress. The variable ω denotes the crack density associated with the family of microcracks oriented in the direction $\vec{n}$, and h is the elastic compliance of crack, respectively defined by:

$$\omega = \frac{Nr^3}{\Omega}, \quad h = \frac{16(1-\nu_0^2)}{3E_0(2-\nu_0)}. \qquad (8)$$

Let $\mathbf{S}^{\text{hom}}$ denote the fourth-order effective elastic compliance tensor of cracked material. Then the free enthalpy function (7) can be rewritten as

$$w_c = \frac{1}{2}\sigma : \mathbf{S}^{\text{hom}} : \sigma .$$

The effective elastic compliance tensor is the expressed as follows:

$$\mathbf{S}^{\text{hom}} = \mathbf{S}^0 + \omega h\{(\vec{n}\otimes\vec{n})\,\overline{\underline{\otimes}}\,\delta + \delta\,\overline{\underline{\otimes}}\,(\vec{n}\otimes\vec{n}) + c(\vec{n}\otimes\vec{n}\otimes\vec{n}\otimes\vec{n})\}, \qquad (9)$$

$c = -\nu_0$ for the opened crack and $c = -2$ for the closed crack. The free enthalpy function (7) and the effective elastic compliance tensor (9) are obtained for the brittle material containing one set of microcracks. This result should be extended to the material containing cracks with arbitrary distributions. This can be done if we assume that there is not any interaction between microcracks. The overall free enthalpy of cracked material is obtained by the addition of the contributions from each set of microcracks. To do this, let us define a continuous crack density function, expressed by $w(\vec{n})$, to represent an arbitrary distribution of microcracks in the space orientation. The macroscopic free enthalpy can be obtained by the integration of function (7) over all the space orientations on the surface of unit sphere denoted by S^2. This surface is decomposed into two complementary but non-overlapped sub-domains, respectively the sub-domain S^{2+} corresponding to the orientations of opened cracks and the sub-domain S^{2-}corresponding to the orientations of closed cracks (PENSÉE et al. [12]). Thus, we have:

$$W_c = \frac{1}{2}\sigma : \mathbf{S}^0 : \sigma + \frac{\eta}{4\pi}\int_{S^{2+}} w(\vec{n})\left(1 - \frac{\nu_0}{2}\right)(\sigma.\vec{n}).<\vec{n}.\sigma.\vec{n}>^+ \vec{n}\,dS$$
$$+ \frac{\eta}{4\pi}\int_{S^2} w(\vec{n})\,\{(\sigma.\sigma):(\vec{n}\otimes\vec{n}) - \sigma:(\vec{n}\otimes\vec{n}\otimes\vec{n}\otimes\vec{n}):\sigma\}\,dS. \qquad (10)$$

In general loading condition, the integral form (10) of the free enthalpy cannot be analytically evaluated. A numerical integration procedure has to be employed. In this paper, a Gauss-type method is chosen for the numerical integration on the surface of the unit sphere (BAZANT and OH [1]). Therefore, the surface of the unit sphere is discretized in a limited number of orientations P. The k^{th} orientation is defined by the unit vector $\vec{n}^k$ and associated with the weight coefficient λ_k. The free enthalpy function (10) is then approximated by:

$$W_c = \frac{1}{2}\sigma : \mathbf{S}^0 : \sigma + \eta\left(1 - \frac{\nu_0}{2}\right)\sum_{k=1}^{P_1} \lambda_k\, w_k(\sigma.\vec{n}^k) < \vec{n}^k.\sigma.\vec{n}^k >^+ \vec{n}^k$$
$$+\eta\sum_{k=1}^{P} \lambda_k w_k\, \{(\sigma.\sigma):(\vec{n}^k\otimes\vec{n}^k) - \sigma:(\vec{n}^k\otimes\vec{n}^k\otimes\vec{n}^k\otimes\vec{n}^k):\sigma\}, \qquad (11)$$

where P_1 denotes the number of orientations corresponding to opened cracks. According to this approximation, the effective elastic compliance tensor (9) can be extended to an arbitrary distribution of microcracks:

$$\mathbf{S}^{\text{hom}} = \mathbf{S}^0 + \eta(2 - \nu_0)\sum_{k=1}^{P_1} \lambda_k w_k(\vec{n}^k\otimes\vec{n}^k\otimes\vec{n}^k\otimes\vec{n}^k)$$
$$+\eta\sum_{k=1}^{P} \lambda_k w_k\, \{(\vec{n}^k\otimes\vec{n}^k)\,\underline{\overline{\otimes}}\,\delta + \delta\,\underline{\overline{\otimes}}\,(\vec{n}^k\otimes\vec{n}^k) - 2(\vec{n}^k\otimes\vec{n}^k\otimes\vec{n}^k\otimes\vec{n}^k)\}. \qquad (12)$$

2.2. CRACK PROPAGATION AND DAMAGE EVOLUTION

In the framework of thermodynamics, the damage evolution law is determined by the formulation of a dissipation potential in the space of the conjugated force associated with the damage tensor. However, the conjugated damage force is usually a complex function of the stresses applied and it is not easy to give a simple physical interpretation. This renders difficult the experimental identification of the damage law. In rock mechanics, as most laboratory tests are performed in stress-controlled or strain-controlled conditions, it appears simpler to formulate the damage evolution law directly in the stress or strain space. Therefore, in the present work, a direct approach is preferred in order to facilitate the experimental determination of the damage evolution law. The damage evolution is directly related to the crack propagation criterion which is based on the fracture mechanics. According to extensive experimental data from triaxial compression tests on brittle rocks (PATERSON [11], WONG [17]), the crack propagation is controlled by both the normal stress and shear stress applied to the crack. The crack growth is caused by increasing shear stress, while the compressive normal pressure has a role of preventing the initiation and growth of microcracks. Different crack propagation criteria can be determined from laboratory data. Based on linear fracture mechanics, the real crack is replaced by a fictitious crack which is subjected to an equivalent tensile force. The fictitious crack is propagating in mode I. The equivalent tensile force is a function of the normal stress and shear stress applied to the real crack. For the sake of simplicity, the following linear function is used in the present work:

$$F(\sigma,\vec{n},r) = \sqrt{r}\,[\sigma_n + f(r)|\vec{\tau}\,|] - C_r \le 0, \tag{13}$$

$$\sigma_n = \vec{n}.\sigma.\vec{n}\,, \quad \vec{\tau} = (\sigma \cdot \vec{n})\cdot(\boldsymbol{\delta} - \vec{n} \otimes \vec{n}), \tag{14}$$

where σ_n is the normal stress applied to crack surfaces, and $\vec{\tau}$ denotes the shear stress vector applied to the crack. This shear stress is generated by the macroscopic deviatoric stress and represents the driving force for the crack propagation. The term with the normal stress allows us to take into account the pressure sensitivity of frictional materials. The parameter C_r denotes the material resistance to crack propagation, which is physically equivalent to the critical toughness (K_{Ic}) in fracture mechanics. $f(r)$ is a scalar valued function controlling the kinetics of crack propagation. Its role is similar to that of the hardening-softening function in plastic models. The expression of this function may be determined from relevant numerical results of micromechanical models and from numerical fitting of experimental data. The general form of the function must, however, satisfy certain requirements. For small crack extents, it should decrease, reflecting the relaxation of local tensile stress as the crack grows away from the source; as the crack length becomes large enough to interact with the stress fields of other nearby cracks $f(r)$ increases or reaches an asymptotic value. The first effect causes initially stable growth and the second marks the onset of accelerated crack interaction producing damage localization and macroscopic failure. The following simple function having these basic features is here used:

$$\begin{cases} f(r) = \eta(\frac{r_f}{r}), & r < r_f, \\ f(r) = \eta, & r \geq r_f, \end{cases} \tag{15}$$

where r_f is the critical crack radius for instable propagation of microcracks, and η is a model parameter. By putting $r = r_0$ in equation (13), we obtain the damage initiation surface in stress space. Similarly, by putting $r = r_f$ in equation (13), the macroscopic failure surface in stress space can be determined. Therefore, the stress levels at the onset of damage growth and at the macroscopic failure state can be entirely determined for any loading paths. The values of the three parameters involved in criterion (13) can be determined from the stress–strain curves obtained from triaxial compression tests. For example, the onset of damage initiation is identified as the point of a stress–strain curve where the linear relationship is lost. The failure state is determined as the pick stress of the stress–strain curves.

3. DETERMINATION OF PERMEABILITY VARIATION

3.1. DAMAGE-INDUCED DILATANCY

According to the closure condition of microcracks (5), the crack is closed under a compressive normal stress. The normal displacement jump vanishes. However, in geomaterials like rocks and concrete, actual crack surfaces are not smooth and contain different kinds of asperities. The roughness of crack surface depends on the microstructure of material (grains and cementation). Due to these asperities, a normal aperture can take place during the relative shear sliding along the crack surfaces. This normal aperture generates a macroscopic volumetric dilatancy, which is commonly observed in brittle geomaterials. Further, in closed cracks, the shear sliding is governed by the local friction law, for instance the Mohr–Coulomb law. The friction law generally induces a hysteretic behaviour during the loading–unloading process. As a consequence of the macroscopic behaviour, hysteretic loops are observed during unloading–reloading cycles. However, this hysteretic phenomenon is not studied in this work.

Let us denote the normal aperture of cracks in the orientation $\vec{n}$ by $e(\vec{n})$. It is a constant (in average sense) for a penny-shaped crack. Therefore, the tensor of damage-related irreversible strains in the constitutive equations can be determined by the integration of normal aperture over all the space orientations:

$$\boldsymbol{\varepsilon} = \mathbb{S}^{\text{hom}} : \sigma + \boldsymbol{\varepsilon}^r, \quad \varepsilon^r = \frac{1}{4\pi}\int_{S^2} \frac{N}{\Omega} e(\vec{n})(\vec{n} \otimes \vec{n})(\pi r^2) dS . \tag{16}$$

The evolution of the normal aperture is associated with the rate of damage evolution. It is assumed that the normal aperture increment is proportional to the increment of average crack radius, that is, $de = c\,dr$, with c being a proportionality coefficient depending on the geometrical roughness of the crack faces. In general, the proportional coefficient c should be a function of damage state. However, in the present work, only a constant value is used as a simplified case of the model.

3.2. ESTIMATION OF THE PERMEABILITY VARIATION

The permeability of a cracked medium is composed of two parts; the initial permeability $\mathbf{k}^0$ due to initial porosity and the crack enhanced permeability $\mathbf{k}^c$. The total permeability is given by $\mathbf{k} = \mathbf{k}^0 + \mathbf{k}^c$. In this work, a simplified case is considered. It is assumed that all cracks are embedded in a porous medium and then connected to the pore networks. In real situations, a certain number of cracks may be hydraulically isolated and do not contribute to the variation of permeability. Therefore, this assumption should lead to an overestimation of the real permeability variation. The crack permeability is essentially due to the crack aperture and evolves with crack propagation. The average crack aperture is associated with crack radius. Therefore, the crack permeability directly depends on the microcrack distribution, which is determined using the anisotropic damage model presented in the previous section. As the microcrack distribution is orientation-dependent, the crack permeability induces an anisotropic character of fluid flow.

Let us consider now a representative volume element (RVE) of rock mass, composed of a porous matrix and the microcracks being distributed at random, subjected to a uniform pressure gradient on the boundary. If all the cracks are fully interconnected to make a flow network, the RVE can be assumed to be a homogeneous, anisotropic porous medium. It obeys Darcy's law, the apparent flow velocity $\vec{v}$ of fluid is related to the macroscopic pressure gradient ∇p through a linking symmetric tensor $\mathbf{k}$ called the permeability tensor:

$$\vec{v} = -\frac{\mathbf{k}}{\mu}\nabla p = -\frac{(\mathbf{k}^0 + \mathbf{k}^c)}{\mu}\nabla p , \tag{17}$$

where μ is the dynamic viscosity of fluid.

The present study is now focusing on the determination of crack permeability. The crack permeability tensor $\mathbf{k}^c$ is regarded as a function of the crack orientation $\vec{n}$, average radius variation $r(\vec{n})$ and average aperture $e(\vec{n})$. For the set of cracks in the given orientation $\vec{n}$, the fluid flow velocity is assumed to be described by the Navier–Stokes equation for laminar flow between two parallel plates:

$$\vec{v}^c(\vec{n}) = -\frac{\lambda}{12}\frac{1}{\mu}e(\vec{n})^2(\delta - \vec{n}\otimes\vec{n})(\nabla p)^c, \tag{18}$$

where $(\nabla p)^c$ is the local pressure gradient applied to the crack. δ denotes the second-order unit tensor. The positive scalar λ, less than the unity, is introduced to take into account the fact that every part of a crack does not work as a conduit. But some parts may be left as dead end. When $\lambda = 1$, the classic cubic law is recovered (SNOW [14]). However, it is important to point out that the use of the Navier–Stokes equation for flow in cracks represents a quite strong assumption. The validity of this equation for fluid flow between rough surfaces of crack is not proved. It is used here for the sake of simplicity because it provides real flow regime with the first approximation. The local pressure gradient may be related to the macroscopic gradient by an appropriate localization law (DORMIEUX and KONDO [5]). In this model, we have used a simplified law by assuming that $(\nabla p)^c = \delta.\nabla p$. This implies that the local pressure gradient is also uniform and equal to the macroscopic one. Therefore, local deviations of pressure gradient are neglected. By analogy to Voigt's bound of elastic compliance tensor of a cracked material, this simplification should correspond to the upper bound of crack permeability.

The macroscopic fluid velocity $\vec{v}$ is determined from the average of local crack velocity $\vec{v}^c$ over the related volume:

$$\vec{v} = -\frac{\mathbf{k}^0}{\mu}\nabla p + \frac{1}{\Omega}\int_{\Omega}\vec{v}^c d\Omega = -\frac{\mathbf{k}^0}{\mu}\nabla p + \frac{1}{\Omega}\int_{\Omega^c}\vec{v}^c d\Omega^c, \tag{19}$$

where Ω^c denotes the volume occupied by the microcracks. According to the anisotropic damage model presented in the previous section, the volume occupied by the set of cracks in the orientation $\vec{n}$ may be expressed by $d\Omega^c(\vec{n}) = N.e(\vec{n}).\pi r(\vec{n})^2$. The total crack volume can be obtained by integration over all the space orientations. Therefore, the macroscopic velocity can be rewritten as:

$$\vec{\mathbf{v}} = -\frac{\mathbf{k}^0}{\mu}\nabla p + \frac{N}{\Omega}\frac{1}{4\pi}\int_{S^2}\vec{v}^c(\vec{n})e(\vec{n})\pi r(\vec{n})^2 dS. \tag{20}$$

Introducing equation (18) into (20), the macroscopic flow velocity is finally expressed by:

$$\vec{\mathbf{v}} = -\frac{\mathbf{k}^0}{\mu}\nabla p + \left(-\frac{1}{\mu}\frac{\lambda}{12}\frac{N}{\Omega}\frac{1}{4\pi}\int_{S^2}e(\vec{n})^3\pi r(\vec{n})^2(\delta - \vec{n}\otimes\vec{n})dS\right).\nabla p. \tag{21}$$

Comparing (21) with the macroscopic Darcy law (17), the macroscopic crack permeability tensor can be determined as follows:

$$\mathbf{k}^c = \frac{\lambda\pi}{12}\frac{N}{\Omega}\frac{1}{4\pi}\int_{S^2} e(\vec{n})^3 r(\vec{n})^2(\delta - \vec{n}\otimes\vec{n})dS\,. \tag{22}$$

By using the same numerical integration method as that used for the calculation of effective elastic compliance tensor, the components of the crack permeability can be approximated by:

$$\mathbf{k}^c = \frac{\lambda\pi}{12}\frac{2N}{\Omega}\sum_{k=1}^{N_g} w_k e(\vec{n}_k)^3 r(\vec{n}_k)^2(\delta - \vec{n}_k\otimes\vec{n}_k)\,. \tag{23}$$

4. NUMERICAL SIMULATIONS

The coupled model proposed contains 9 parameters, which can be determined from a series of triaxial compression tests with different confining pressures. The initial elastic constants of intact material, i.e. E_0 and ν_0, are determined from the linear part of stress–strain curves. The parameters involved in the crack propagation criterion, i.e. r_0, r_f, η and C_r, can be identified drawing the damage initiation surface (initial yield surface) for $r = r_0$ and the failure surface for $r = r_f$ in the conventional p–q stress plane. The damage initiation surface is determined from the stress level where the linearity is lost, while the failure surface is obtained from the peak stresses. The normal dilation parameter χ and the crack number involved in the RVE can be estimated from the non-linear responses of the axial and radial strains during a triaxial compression test. Finally, the roughness coefficient of crack faces λ can be determined from experimental data on the increase of permeability during a triaxial compression test. The model proposed is applied to a typical brittle rock, sandstone. For this material, the typical values of model parameters are as follows:

$$E_0 = 20300\ \text{MPa}\,,\ \nu_0 = 0.26\,,$$

$$r_0 = 3\times 10^{-3}\,\text{m}\,,\ r_f = 9\times 10^{-3}\,\text{m}\,,\ C_r = 1.06\ \text{MPa}\sqrt{\text{m}}\,,\ \eta = 9.75\times 10^{-3}\,,$$

$$N = 6.3\times 10^6\,,\ \chi = 0.0005\,,\ \lambda = 0.083\,.$$

Figures 1 and 2 show the simulation of two triaxial compression tests. There is a good agreement between the numerical simulations and experimental data. The anisotropic damage proposed describes the main features of mechanical behaviours of typical brittle rocks such as non-linearity, volumetric dilatancy and pressure dependence. For the determination of the variation of permeability due to crack growth, an isotropic initial permeability is assumed. Further, in each orientation, the number of microcracks remains the same, but the average crack radius is different. The average radius of cracks in each orientation is explicitly determined by the propagation crite-

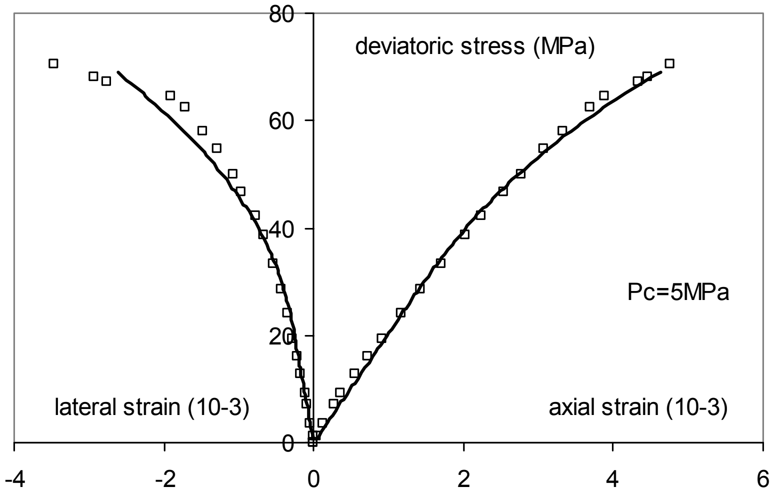

Fig. 1. Simulation of a triaxial compression test at 5 MPa confining pressure (the continuous lines are numerical simulations)

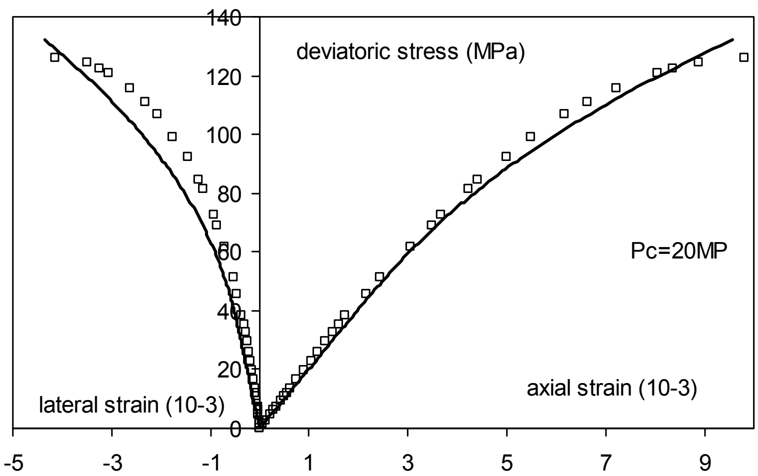

Fig. 2. Simulation of a triaxial compression test at 20 MPa confining pressure (the continuous lines are numerical simulations)

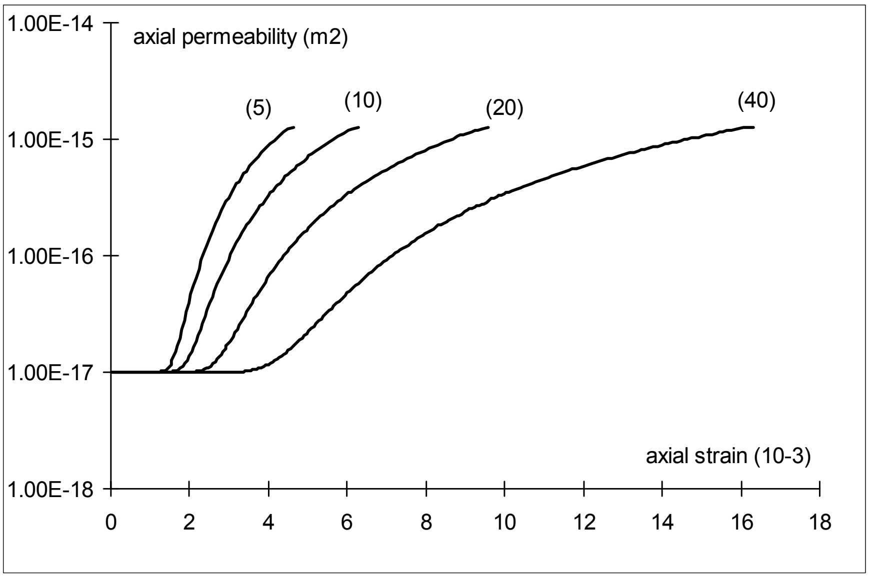

Fig. 3. Variation of the axial permeability due to crack propagation in triaxial compression tests at different confining pressures

rion, and the average normal aperture is determined using the dilatancy coefficient. Numerical predictions of the variations of permeability in the axial direction during a triaxial compression test are shown in figure 3 for different confining pressures. Unfortunately, experimental data on the permeability variation are not available for this rock under such test conditions. It is then impossible to give a quantitative comparison. However, from the qualitative point of view, these are qualitatively in agreement with experimental data obtained in brittle rock materials, mentioned in the first part of the paper.

5. CONCLUSIONS

An anisotropic damage model is proposed by taking into account the variation of permeability due to growth of microcracks. The formulation of the model is based on micromechanical analysis and experimental evidences relating to brittle materials like rocks and concrete. The damage evolution is determined from the crack propagation condition. By assuming fully connected microcracks, the permeability variation due to crack growth is explicitly coupled with the evolution of mechanical damage of material. The roughness of crack faces is taken into account. The model proposed is able to describe the main features of mechanical behaviours of brittle materials and the coupling with hydraulic flow. The simulations given by the model proposed are qualitatively in agreement with experimental data. However, it is a progressing but promising work, extensive experimental validation will be necessary to check the performance of the model. Some extensions could also be introduced, for example, considering a partial crack connectivity, to improve the performance of the model.

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