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TECHNICAL NOTE

THE NON-LINEAR MOHR–COULOMB MODEL FOR SANDS

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1. INTRODUCTION

An initial relative density and stress level greatly affect the maximum shear strength of sand. High relative density at low effective stress involves a dilative behaviour under shear and leads to a shear strength much greater than that obtained under shear of loose sand at high stress level. This behaviour has been widely studied in literature (BOLTON [1], CORNFORTH [2], SCHANZ and VERMEER [4], SHIMOBE and MIYAMORI [5]) and can be simply described by non-linear perfectly plastic Mohr–Coulomb model presented in this paper. The perfectly plastic model is a constitutive model with a fixed yield surface in stress space fully defined by model parameters and not affected by plastic strains. In the stress states, represented by points within the yield surface, the sand behaviour is purely elastic and the stresses are reversible. Generally, in isotropic geomaterials, the Mohr–Coulomb model is described by five parameters: E – Young's modulus, ν – Poisson's ratio, Φ – an internal friction angle, c – the cohesion and Ψ – the dilatancy angle. Generally, a sand is treated as a cohesionless material, where c = 0. In the classical Mohr–Coulomb model, the friction and dilatancy angles are constant material parameters in the non-linear model presented, whose parameters depend on an initial relative density and stress level of sand.

2. SHEAR STRENGTH AND DILATANCY OF SANDS

Two parameters are used to describe the elastic behaviour of sands. The Poisson ratio ν is a constant and ranges from 0.15 to 0.35. The Young modulus for silica sand is the function of an effective hydrostatic pressure p' (RICHART at al. [3])

$$E = 2\ (1+\nu)\, G_o\ p_a \frac{(2.97-e)^2}{1+e} \left(\frac{p'}{p_a}\right)^{0.5}, \tag{1}$$

where p_a is the atmospheric pressure, e – the void ratio and G_o – a material parameter ranging from 35 to 65.

According to BOLTON [1], the effective maximum friction angle Φ'_m in triaxial and biaxial compression is related to the critical friction angle Φ'_{cv} and a relative dilatancy index I_R. Hence we have

- for triaxial compression

$$\Phi'^{tx}_m - \Phi'_{cv} = 3\, I_R\,, \tag{2}$$

- for biaxial compression

$$\Phi'^{bx}_m - \Phi'_{cv} = 5\, I_R\,. \tag{3}$$

The relative dilatancy index is as follows:

$$I_R = I_D\,\{Q - \ln(\,p'/p'_{\min})\} - R \quad \text{for} \quad p' \geq p'_{\min} \tag{4}$$

and

$$I_R = Q I_D - R \quad \text{for} \quad p' \geq p'_{\min}\,, \tag{5}$$

where I_D is the initial relative density of sand. For silica sand $p'_{\min}$ = 150 kPa, Q = 5 and R = 1.

The maximum dilation rate in failure state is

$$\left(-\frac{d\varepsilon_{\mathrm{v}}}{d\varepsilon_1}\right)_m = 0.3\ I_R\,, \tag{6}$$

where ε_1 and ε_v are, respectively, the axial and volumetric strains. The dilation rate expressed by equation (6) is valid for triaxial and biaxial compression conditions.

SCHANZ and VERMEER [4] showed that the dilatancy angle Ψ defined by equation

$$\sin\Psi = -\ \frac{d\,\varepsilon_v/d\,\varepsilon_1}{2 - d\,\varepsilon_v/d\,\varepsilon_1} \tag{7}$$

may be expressed in the form

$$\sin\Psi = \frac{0.3\, I_R}{2 + 0.3\, I_R}\,. \tag{8}$$

3. THE NON-LINEAR MOHR–COULOMB CRITERION

The non-linear failure criterion for sands can be defined on the basis of the maximum mobilized friction angle Φ'_m, which is a function of the critical angle

Φ'_{cv}, the initial relative density I_D and the mean effective stress p'. For $p'_{\min} \le p' < p'_{cr}$

$$\Phi'_m = \Phi'_m\,(\Phi'_{cv}, p', I_D)\,, \tag{9a}$$

for $p' < p'_{\min}$

$$\Phi'_m = \Phi'_m\,(\Phi'_{cv}, I_D)\,, \tag{9b}$$

and for $p' \ge p'_{cr}$.

$$\Phi'_m = \Phi'_{cv}\,. \tag{9c}$$

Beyond p'_{cr} the behaviour of the sand is non-dilatational. According to equation (6) for $p' > p'_{cr}$, $I_R = 0$ and p'_{cr} can be expressed by the following equation (figure 1)

$$p'_{cr} = \exp\left(10 - \frac{1}{I_D}\right). \tag{10}$$

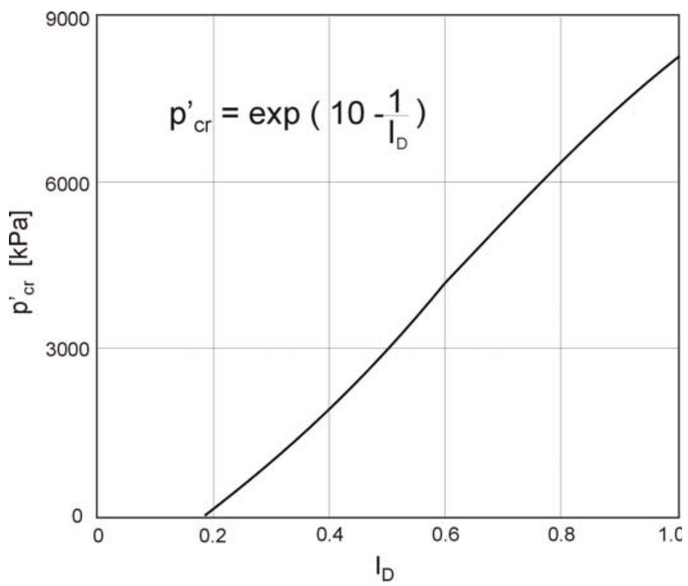

Fig. 1. The dependence p'_{cr} on I_D for silica sand

The failure criterion is defined for $I_R > 0$ $(I_D > 0.2)$.

In figure 2 the dependence of I_R on $p'/p'_{\min}$ for $p'/p'_{\min} > 1$ is shown. Generally, it is seen that $0 \le I_R \le 4$ for silica sand.

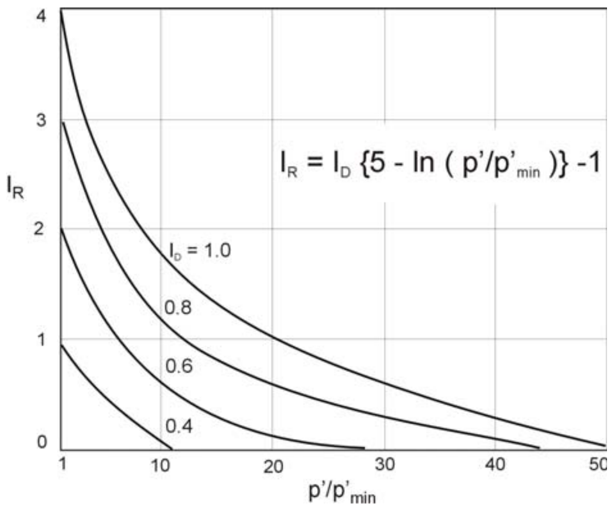

Fig. 2. The dependence I_R on $p'/p'_{\min}$

Taking account of a general three-dimensional effective stress state, the non-linear Mohr–Coulomb criterion can be expressed as follows:

$$F = (\sigma_1' - \sigma_3') - (\sigma_1' + \sigma_3')\sin\Phi_m' = 0\,, \tag{11a}$$

or

$$F = p'\sin\Phi_m' + \sqrt{J_2}\left(\cos\theta - \frac{\sin\theta\sin\Phi_m'}{\sqrt{3}}\right) = 0 \tag{11b}$$

if $p' < p'_{cr}$ and

$$F = \Phi_m' - \Phi_{cv}' = 0\,, \tag{11c}$$

or

$$F = p'\sin\Phi_{cv}' + \sqrt{J_2}\left(\cos\theta - \frac{\sin\theta\sin\Phi_{cv}'}{\sqrt{3}}\right) = 0 \tag{11d}$$

if $p' > p'_{cr}$.

In equations (11a) and (11b)

$$\Phi_m' = \Phi_m'^{tx} + (\Phi_m'^{bx} - \Phi_m'^{tx})\sin\left\{3\left(\theta + \frac{\pi}{6}\right)\right\}, \tag{12a}$$

where

$$J_2 = \frac{I_1^2 - 3I_2}{3}\,, \tag{12b}$$

$$\theta = \frac{1}{3} \arcsin \frac{-3\sqrt{3}\, J_3}{2\, J_2 \sqrt{J_2}} , \tag{12c}$$

$$J_3 = \frac{2\, I_1^3 - 9\, I_1\, I_2 + 27\, I_3}{27} \tag{12d}$$

and

$$I_1 = \sigma_1' + \sigma_2' + \sigma_3' = 3p' \ , \tag{13a}$$

$$I_2 = \sigma_1'\sigma_2' + \sigma_1'\sigma_3' + \sigma_2'\sigma_3' \ , \tag{13b}$$

$$I_3 = \sigma_2'\sigma_2'\sigma_3' \ . \tag{13c}$$

Figure 3 shows the failure surface expressed by equations (11a) and (11b) in the principal stress space.

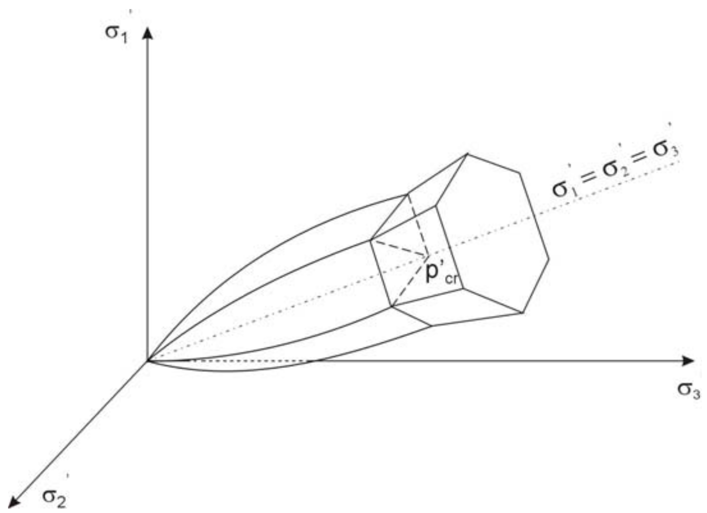

Fig. 3. The non-linear Mohr–Coulomb criterion in principal stress space

4. THE PLASTIC POTENTIAL FUNCTION

A non-associated flow rule based on the following plastic potential function

$$G = p' \sin\Psi + \sqrt{J_2} \left(\cos\theta + \frac{\sin\theta \sin\Psi}{\sqrt{3}} \right) = 0 \tag{14}$$

is assumed in this paper. Assuming

$$\Psi = \Psi^* = \arcsin 3I_R \tag{15}$$

the plastic potential function (14) is identical to a plastic potential function defined by SIMONINI [6]. Figure 4 shows the dependence of Ψ and Ψ^* on I_R.

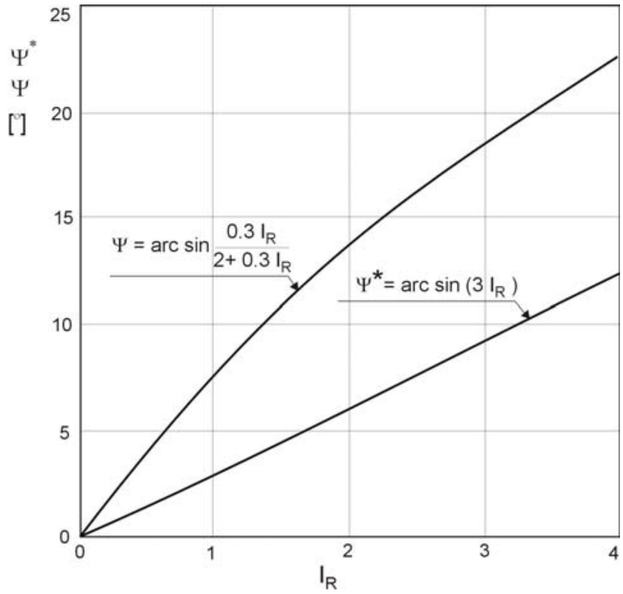

Fig. 4. The dilatancy angle as a function of I_R

The values of Ψ are about twice as high as the values of Ψ^*.

5. CONCLUSIONS

The behaviour of silica sand can be simply described by the non-linear Mohr–Coulomb model presented in the paper. The shear strength, secant angle of internal friction in failure state, is related to the angle of friction in critical state, stress level, initial relative density and type of deformation. Nowadays that rate of dilation, the value of angle of dilation, is related to an initial relative density and stress level, but not related to the type of deformation generally accepted. The constant values of Poisson's ratio and Young's modulus as the functions of stress level are suggested in this model.

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