

# EVALUATION OF GEOTECHNICAL PARAMETERS IN MODERN LABORATORY TESTS ACCOUNTING FOR LOADING PATHS

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**Abstract:** Soil response to loading depends on subsoil history and boundary conditions of the analysed problem. Hence the conclusion that parameters of soil constitutive models are also dependent on these factors, especially when simple models are considered. The paper presents a new approach to parameters evaluation, called Loading Path Method, based on Stress Path Method by Lambe and Strain Path Method by Baligh. The problems connected with the new method are discussed.

## 1. INTRODUCTION

The leading role of penetration tests, such as CPTU, SCPTU or DMT, in estimating geotechnical parameters is beyond dispute. This superiority arises from extraordinary practical facilities of the above methods: profiling ability, high penetration rate and relatively low cost of measurements.

Laboratory investigations, such as conventional axisymmetric or true triaxial tests, are slow, expensive and limited to single points of soil profiles, and therefore do not compete with present-day in situ techniques. This does not minimize the importance of the laboratory tests, whose most specific virtue is that both the strain and effective stress fields within the sample volume are approximately homogeneous. Thus, no back analysis of the boundary value problem is needed for interpreting measurements in order to estimate deformation and strength characteristics. Other advantages of modern laboratory tests lie in the control of loading path, internal measurements of microdisplacements and the use of bender elements. All the above features make them the most valuable source of the database of correlations between the CPTU, SCPTU or DMT indicators (normalized cone resistance, etc.) and mechanical soil parameters. To obtain parameter estimations, which are as reliable and exact as possible, complementary triaxial tests of high quality are carried out on undisturbed soil samples. This is particularly justified in the case of weak soil layers which essentially influence foundation settlement, slope stability or earth pressure.

Perhaps the most important condition of reliable parameter identification is the maximally faithful simulation of geotechnical and recent loading histories at points of sampling within laboratory procedure. This is caused by strong soil sensitivity to effective stress or strain paths [4], [7], [8].

The research concept sketched above was originated independently by LAMBE [9] and DAVIS and POULOS [2] as the Stress Path Method. BALIGH [1] developed the in-

verse one, called the Strain Path Method. They are successfully used around the world to the present day and there are no reasons to abandon back from the original ideas. However, the details of procedures leave room for improvement, as based on the simplified assumptions:

1. A full effective stress control in the so-called problems of shallow soil mechanics and full strain control in the so-called problems of deep soil mechanics (BALIGH [1]).

2. The simulation of any true triaxial effective stress or strain path in the axisymmetric compression plane.

3. The starting point in analysis is fitting the theoretical and experimental values of the effective stress or strain components considered to be the response to the end point of loading path.

The above standard simplifications can sometimes appear to be far from reality. A new research concept, called the Loading Path Method, originated in earlier authors' work [5], [6], is presented here more synthetically. This method being a generalization of both the Stress Path Method and the Strain Path Method dispels doubts and overcomes inadequacies included in the assumptions mentioned.

## 2. ANALYSIS OF LOADING PATHS

A starting point of every loading path must correspond to the in situ conditions before taking the sample. These conditions are identified by the initial (in situ) effective stress components (1) referred to the cylindrical coordinate system  $(r, \theta, z)$  with vertical  $z$ -axis:

$$\sigma'_{r0} = \sigma'_{\theta0} = K_0 \gamma z. \quad (1)$$

In equation (1),  $\gamma$  denotes the average overburden unit weight and  $K_0$  is the earth pressure at rest, which can be evaluated on the basis of a special oedometer test or one of the in situ penetration tests. Rough estimation can be performed using the generally known empirical formulas including the overconsolidation ratio OCR.

In the Cambridge notation of stress invariants, equation (1) takes the form (2) [3], where  $p'_0, q'_0, \Theta_0$  are the initial values of the effective mean stress, the stress intensity and Lode's angle:

$$p'_0 = \frac{1}{3} \gamma z (1 + 2K_0), \quad q'_0 = \gamma z (1 - K_0), \quad \Theta_0 = -\frac{\pi}{6}. \quad (2)$$

It should be strived for a loading path as close as possible to the real behaviour of the subsoil at the point under analysis. Unfortunately, obtaining such an information is impossible under field conditions; the only solution is a computer-aided simulation of the problem. In a program based on finite element method (FEM), a foundation-subsoil system with a full exploitation history is created. Taking into account a subsoil profile

and a possibility of using a constitutive model more sophisticated and adequate than the elastic one, provided that it is built in the computer database, a superiority of such an approach is apparent.

The shape of the loading path depends upon the choice of the constitutive model. As an example, let us analyse a cylindrical tank, heavily loaded, founded on a normally consolidated layered subsoil (figure 1). Points 164 and 170 are located in a layer of organic clay with MCC parameters:  $M = 1$ ,  $\lambda = 0.15$ ,  $\kappa = 0.015$  and the corresponding C–M parameters assessed as:  $c = 0$  kPa,  $\phi = 25.5^\circ$ ,  $E = 3$  MPa;  $\nu$  value was assumed on the level of 0.3.

The difference in the shape of the  $p$ – $q$  stress paths at the points below the centre and the wall of the tank for Modified Cam Clay (MCC) and Coulomb–Mohr (C–M) models is very distinct. In the time of unloading, caused by excavation and reloading to the initial value of stresses, both paths coincide. It is a result of a very simplifying assumption in MCC that inside the yield surface the soil behaviour is elastic as in C–M model. Only just after reaching the yield surface (at the point of initial stress) the stress path of the MCC model departs into hardening, while the stress path of the C–M model remains without any substantial change in direction. Limit state is reached at the point 170 for C–M model, while stress states calculated by means of MCC model at both points are far from critical state.

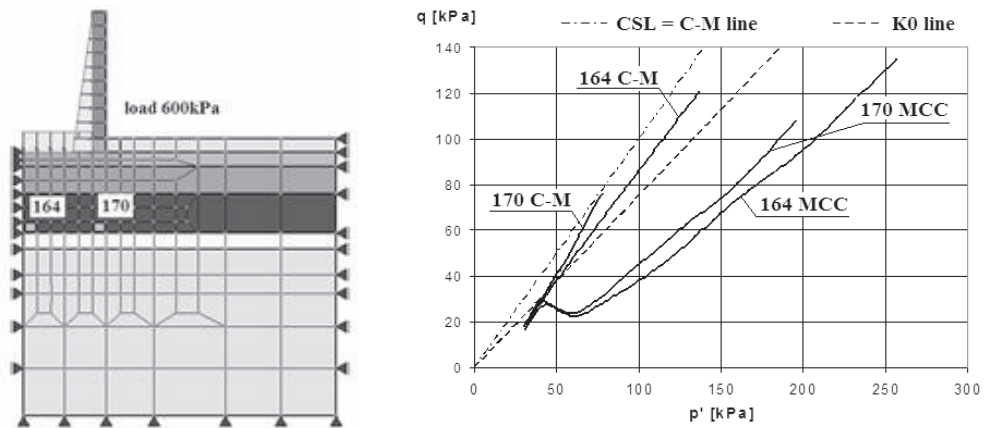


Fig. 1. Cylindrical tank–subsoil system: FEM mesh. Stress paths at two points for Modified Cam Clay model (MCC) and Coulomb–Mohr model (C–M);  $K_0$  – coefficient of earth pressure at rest, CSL – critical state line, C–M line – Coulomb–Mohr limit state line

To carry out an exact quantitative comparison of the stress paths for the C–M and MCC models, it would be necessary to evaluate the parameters  $E$ ,  $\phi$  and  $c$  and  $\lambda$ ,  $\kappa$ ,  $M$ ,  $G$  for the same soil. Unfortunately, only some of them are interrelated, e.g., in over-consolidation state, equations (3) would be valid:

$$c = \frac{3 - \sin \phi}{6 \cos \phi} \left( M - \frac{6 \sin \phi}{3 - \sin \phi} \right) \frac{OCR \gamma z}{2} \left[ \frac{1 + 2K_0^{NC}}{3} + \frac{3(1 + K_0^{NC})^2}{M^2(1 + 2K_0^{NC})} \right], \quad (3)$$

$$E = \frac{3(1 - 2\nu)(1 + e_0)}{\kappa} \gamma z.$$

Naturally, in the stage of FEM analysis one may only assume the parameters' values and the purpose of the whole algorithm is to estimate their final proper values. That is why an iterative procedure should be implemented. It is a time-consuming task, and therefore confining oneself to only one correction seems reasonable.

There is still a problem of the loading identification. In the case of the structure–soil interaction, neither stress nor strain states are fully controlled. It must be decided which of them would become the load and which one – the response. Various criteria may be considered. In the authors' opinion, a reasonable concept is to assume that just in the loading state there are smaller average relative discrepancies in paths for different constitutive models. For various geotechnical situations this can be the stress state, the strain state or the mixed one. However, the decision is often affected by the pragmatics of the laboratory test: the standard of the apparatus equipment, the test speed, etc.

### 3. CAPABILITIES OF MODERN LABORATORY APPARATUSES IN THE ASPECT OF A LOADING PATH'S SIMULATION

A conventional triaxial apparatus is a basic laboratory equipment enabling experiments according to loading paths. Despite its many advantages and continuous improvement of its construction, the apparatus suffers from one fundamental disadvantage: it enables testing only in axisymmetrical state of loading. In the 3D space  $p$ – $q$ – $\Theta$ , this is tantamount to a limitation of  $\Theta$ 's value to  $-\pi/6$  and a limitation of the case studies analysed to axisymmetrical ones with negligible tangent stresses. Such stress conditions take place only at the points strictly below the centre of a circular foundation.

Simulation of a stress path at another point of subsoil in a conventional triaxial apparatus would be connected with such a simplification that allows neglecting the tangent stresses and averaging the normal horizontal stresses. Such a treatment, however, influences directly the shape of a stress path, first of all – the value of a stress deviator.

It should be stressed that a continuous control of the stress state, even axisymmetrical, is possible only in triaxial apparatus equipped with the system of automatic control. Within this system the piston is being displaced using an additional chamber with controlled internal pressure. These conditions are fulfilled, e.g., in an apparatus constructed by Bishop and Wesley.

The most common triaxial apparatuses constructed by Bishop and Henkel in practice enable only manual control of a water pressure  $\sigma_r$  in the chamber and the speed  $\varepsilon_a$

of the piston displacement. Using such an apparatus only allows imposing “mixed conditions”  $\sigma_r$ - $\varepsilon_a$  in finite increments.

Simulation of loading paths with different values of stress or strain in both horizontal directions ( $\sigma_2 \neq \sigma_3$  or  $\sigma_r \neq \sigma_\theta$ ,  $\varepsilon_2 \neq \varepsilon_3$  or  $\varepsilon_a \neq \varepsilon_\theta$ ) is possible in the so-called true triaxial apparatus. Advanced apparatuses of this kind enable testing the unsaturated samples with the control of water and air pore pressure, which is essential in the case of undisturbed samples. Unfortunately, the apparatus does not allow any control of principal stresses’ directions yet.

Full control of magnitudes and directions of principal stresses may be obtained only in a hollow cylinder apparatus. The device is commonly used in modern laboratories to examine the influence of inherent and induced anisotropies, mean principal stress  $\sigma_2$  and rotation of principal stresses on soil response. The principal stresses are imposed not directly, but by the combination of a vertical axial force, a torque, an internal and external cylinder pressure. Disadvantages of the apparatus are as follows: no strain control and a complicated testing procedure.

#### 4. ESTIMATION OF PARAMETERS

Estimation of parameters of any constitutive model of soil by means of the Loading Path Method is based on a back analysis. If the loading path and constitutive equations of the calibrated model are known, it is possible to calculate the theoretical response of soil. Next, by changing parameters, the shape of theoretical curve may be fitted to the response obtained in laboratory test.

We deal with the simplest case if the stress state  $p$ - $q$  is under control and strain state  $\varepsilon_r$ - $\varepsilon_s$  makes up the response path, or vice versa, these are the invariants which define the majority of models. The case of mixed conditions, when loading is defined as, e.g., the chamber pressure  $\sigma_r$  and the velocity of axial displacement  $\varepsilon_a$ , is more complicated, since the constitutive equations assume an indirect form.

Searching for the parameters of a specified model may be carried out by means of the simplest methods like Systematic Searching or Monte Carlo, but when for model calibration more than two parameters are indispensable, these methods become ineffective. Optimization procedures, e.g., Pattern Search, Simplex, Complex or popular Genetic Algorithms, are then worth using.

Independently of choosing the method, some criterion of fitting must be set. This will be the formulation of optimization problem.

Let us assume that a loading path is a curve  $p$ - $q$ . If we want to determine the parameters which characterize best the soil behaviour throughout loading, the response path, both determined in laboratory and theoretically, should be defined in the complementary space of the dependent variables as a strain path  $\varepsilon_r$ - $\varepsilon_s$ . Under some conditions, we might also consider a simplified response with one unknown only, e.g.,  $p$ '- $\varepsilon_v$ .

or  $q-\varepsilon_s$ . In such a case, one should not, however, expect a result in the form of a full set of model parameters, since as a result of the limitation some parameters may not take any part in the optimization process.

The criterion of fitting may be the finding of a response path for the set of optimal parameters which fits a whole experimental curve. If we choose a coincidence of a “full” dependent response, here  $\varepsilon_v-\varepsilon_s$  as the criterion of fitting, then the simplest solution can be the minimum of objective function in the form of the sum of absolute distances of all corresponding points of experimental and theoretical response curves:

$$R = \sum_i^n \sqrt{\left(\frac{x_i^T - x_i^E}{\Delta x}\right)^2 + \left(\frac{y_i^T - y_i^E}{\Delta y}\right)^2}, \quad (4)$$

where:

$x_i^T, y_i^T$  – the theoretical response values,

$x_i^E, y_i^E$  – the experimental response values,

$\Delta x = \max|(x_i^T, x_i^E)| - \min|(x_i^T, x_i^E)|$ ,

$\Delta y = \max|(y_i^T, y_i^E)| - \min|(y_i^T, y_i^E)|$ ,

$n$  – the number of measurements.

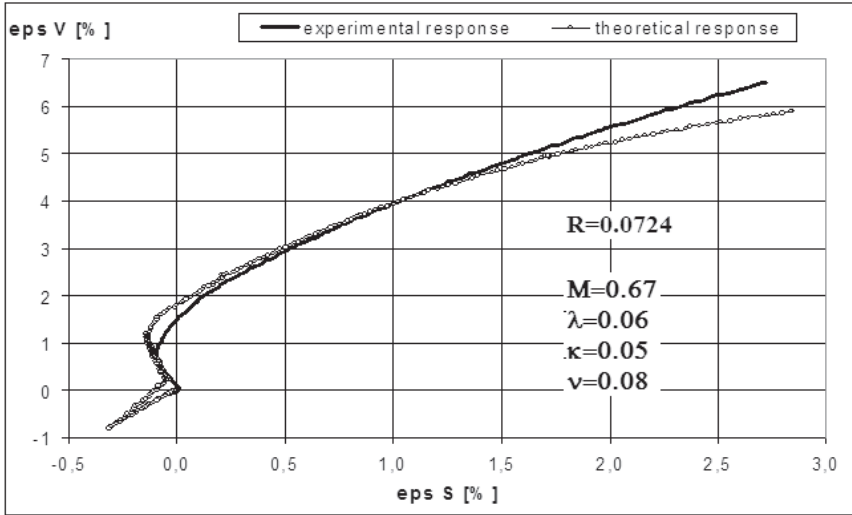


Fig. 2. The experimental and the best theoretical MCC responses to the path  $p-q$  at point 164 given in figure 1

In formula (4) the denominator has been introduced in order to make  $R$  independent of the range of the values being compared (in the case of estimating parameters on

the basis of comparison of, e.g., response paths in the space  $\sigma_a-\varepsilon_r$ , stresses may range, e.g., from 0 to 300 kPa, while strains, e.g., from 0 to 0.3). Dividing the distance of stresses and strains by their range causes that finally dimensionless numbers in the range of 0–1 are summed.

An example of fitting experimental and theoretical curves, being a response to the stress path  $p-q$  at the point 164 for MCC model, is presented in figure 2. The smallest distance between both curves was obtained at the parameters  $M = 0.67$ ,  $\lambda = 0.06$ ,  $\kappa = 0.05$  and  $\nu = 0.08$ . The objective function  $R$  (formula (4)) equals then 0.0724.

If we were interested in the final magnitude of strain only, the criterion of fitting could be just the comparison of the last readings.

## 5. SUMMARY

Summarizing all the above considerations leads to the following conclusions:

1. Laboratory tests are a necessary and important complement to penetration tests, on condition that the influence of paths is considered in the most appropriate way.
2. Assumptions of the classical stress or strain path methods concerning the determination of loading paths are unjustified – there is no strict control of either strains or stresses.
3. The adequacy of loading paths and response paths depends on the abilities of testing equipment.
4. Geotechnical parameters are estimated on a basis of the best fitting of theoretical and experimental response paths with use of available methods of regression analyses.

The problem of model parameters, a key geotechnical problem, in spite of the dynamic development of testing methods remains still unsolved and creates a field for extensive research.

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