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THE ROLE OF A MOVABLE SANDY-BED IN MODELLING OPEN-CHANNEL FLOW

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Abstract: The sediment transport mechanisms in rivers have recently attracted the attention of civil engineers and decision-makers due to its importance for environmental issues and water resources management. The complexity of this phenomenon arises from several assumptions for its theoretical analysis, such as considering the water density as a constant with the value of 1000 m^3/kg. A movable sandy bed determines the variation of the fluid's density because of the sedimentation processes (e.g., lifting forces) that take place at the boundary between the channel bed and the fluid in motion. When the bed's material contains mainly coarse soils, such as sand or gravel, the bed constitutes a porous medium where the water is flowing as well.

This paper describes some experimental investigations on how the water density varies with the depth, velocity and channel roughness under established hydrodynamic conditions. In addition, the velocity at the boundary between the bed and the water is estimated using a numerical model which calculates the flow through the sandy layer. The assumptions that WANG and WU [7] considered as the most commonly adopted in the models for sediment transport and free surface flow are discussed.

1. INTRODUCTION

Sedimentation processes in rivers are considered very complex phenomena that have been investigated since the beginning of the civilisation in order to improve the quality of life of the people living close to streams, reservoirs and rivers. The material that is transported by the river flow not only affects the geometry of the cross sections along the waterways (classical sediment transport), but also the hydrodynamic conditions of the fluid in motion that becomes a mixture of clean water and suspended sediment. The suspended load brings about the changes of the fluid density that normally is taken as constant for hydrodynamic modelling with the value $\rho = 1000$ kg/m^3. Hence, the kinematic viscosity that is directly related to the density cannot be considered as constant.

When the upper layer of any river bed is a porous medium, e.g., when the river bed is made of gravel or coarse material combined with sands and fine soils, the water depth of the channel represents a hydraulic head that can derive in flow through this porous medium. Thus, a velocity field exists within the river bed and it can be estimated to check whether the velocity of the water in the boundary is significant or can be neglected for modelling purposes. This paper describes the classical approaches to model free surface flow and sediment transport in open channels highlighting the importance of the soil–water interaction. The following cases are described: the variation of the liquid density with the water depth and velocity; the calculation of the water velocity at the boundary between the fluid in motion and the channel bed and the differences between the results of a popular unidimensional program and the measurements of a physical model in a laboratory.

2. MODELLING FLOW AND SEDIMENT TRANSPORT IN OPEN CHANNELS

Several models and methodologies have been developed in order to simulate flow and sediment transport mechanisms. The simplest methodologies were based on field observations and physical modelling in a laboratory; nowadays, more complex models are developed on the basis of numerical and mathematical computations. Free surface flow and sediment transport are characterized by turbulence, free-surface variation, bed change, etc. Thus, most of the models adopt several assumptions which simplify the theoretical analysis. WANG and WU [7] summarized three general assumptions as follows:

a) The interaction between flow and sediment movement can be neglected because the sediment concentration is low.

b) The flow can be calculated assuming a fixed bed because the bed change is much slower than the flow movement.

c) The interactions among different size classes of moving sediments are ignored.

These assumptions will be mentioned in the third and fifth sections of this paper, where the validation of these assumptions will be discussed.

The simplest equation to model free surface flow is *the equation of motion for a stream point following a trajectory s*, better known as the equation of conservation of energy (SOTELO-ÁVILA [5]); its differential form is as follows:

$$\frac{\partial E}{\partial s} = \frac{\partial}{\partial s}\left(z + \frac{p}{\gamma} + \frac{v^2}{2g} + h_r \right) = -\frac{1}{g}\frac{\partial v}{\partial t}, \tag{1}$$

where z is the elevation of the liquid vain with respect to an established datum, p is the pressure acting on it, γ is the specific volume of the fluid, v is the velocity of the vain, h_r is a coefficient that represents the loss of energy and g is gravitational acceleration. The specific weight (γ) is the product of the density and the gravitational acceleration, thus if the density varies with z, the specific volume cannot be considered as constant. This equation neglects the viscous effects of the fluid in motion; nevertheless it is commonly used as the basis of one-dimensional models including HEC-RAS.

The modelling of sediment transport should consider the viscous effects; several 1D models were developed based on the St. Venant equations, while in 2D and 3D models, the Reynolds average (RANS) and the Navier–Stokes equations are the governing equations. Derived from the Newton's second law of motion, the Navier–Stokes set of equations is the basis for modelling viscous flow. The general form of the set of equations can be written as follows:

$$\frac{\partial U_i}{\partial t}+U_j\frac{\partial U_i}{\partial x_j}=\frac{1}{\rho}\frac{\partial}{\partial x_j}\left(-P\delta_{ij}+\rho\upsilon\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)\right), \qquad (2)$$

where U is the velocity vector; ρ stands for the density of the viscous fluid; υ is the kinematic viscosity and P is the pressure acting on the system. The Navier–Stokes equations are the basis of the majority of hydrodynamic models; nevertheless and due to the fact that natural streams, channels and rivers flow invariably in turbulent regime, the validity of the Navier–Stokes equations is limited. Highlighting that the density and viscosity are to be defined for modelling purposes, the assumption that these two parameters are constant is not valid for multiphase flow or when there are external agents that can modify these parameters.

The Navier–Stokes equation is commonly accompanied by the continuity equation (which will be mentioned in the fifth section) in order to guarantee the incompressibility of the fluid in motion, which in the case of a river is a mixture of water and the material transported from the channel bed.

3. THE VARIATION OF THE PROPERTIES OF THE FLUID IN MOTION DUE SHEAR STRESSES

The figure 1 depicts the traditional velocity profile for open channels. In this figure, there is shown as well the scientific classification of flow layers according to the kind of shear stresses and flow type that take place within the fluid in motion. Two kinds of shear stresses are identified: the viscous and the turbulent shear.

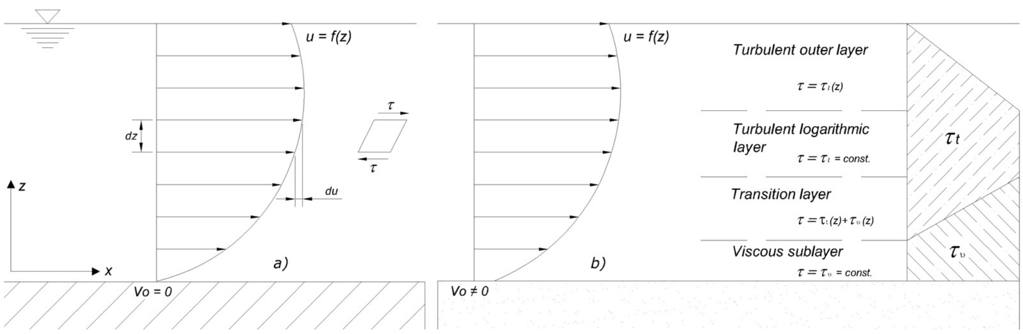

Fig. 1. The velocity profile $u = f(z)$ and the classification of flow layers

Close to the river bed, where the flow tends to be laminar, a viscous shear stress can be defined using the Newton's law of viscosity. In addition, and considering that the majority of flows in nature are turbulent, a typical phenomenon occurs over the water depth, namely the fluctuations of velocity denoted by u' and w' (LIU [2]) in the x and z directions, respectively. This fluctuation of velocities causes turbulent shear stresses on the basis of the Prandtl's mixing length theory. The sum of both stresses is expressed by

$$\tau = \tau_\upsilon + \tau_t = \rho\upsilon\frac{du}{dz} + (-\rho\overline{u'w'}), \tag{3}$$

where τ_υ is the viscous shear stress, τ_t is the turbulent shear, $\overline{u'w'}$ is the velocity fluctuation vector. Shear stresses acting on the area of the material from the bed generate the forces that drag or lift the solid particles. Due to the lifting forces acting on the bed the finest material and the part of the coarse material will be transported as suspended load. This lifting force F_L is defined by:

$$F_L = \frac{1}{2}\rho C_L A U^2, \tag{4}$$

where the C_L is the lift coefficient, which depends on the shape and surface roughness of the body as well as on the Reynolds number, A is the projected area of the grain body perpendicular to the plane of the flow direction, and U is the average velocity of the flow in steady state. The shear stresses and lifting forces acting on the bed material can change the value of the density and viscosity with the water depth. The concentration of suspended sediment has been a well-researched topic. Nevertheless, the relation that describes the variation of the water density when it is affected by the bed material in suspension has not been derived.

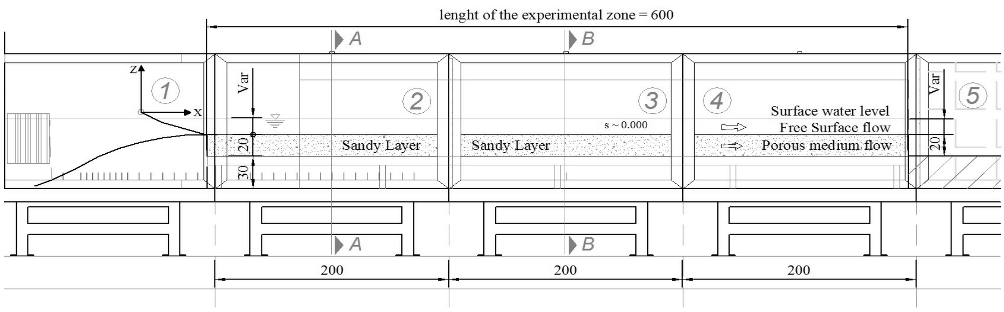

Fig. 2. The flume in the open air laboratory

Hence, a series of experimental tests were carried out in the laboratory of the WUT, Institute of Geotechnics and Hydroengineering, to verify whether or not there is a significant variation of this water property with the depth and flow velocity. The open air flume shown in figure 2 (6.0 m long, 0.5 m width, and 1.0 m high) was used for this purpose; this channel was provided with a special structure to allocate a 0.20 m sandy bed in order to observe and measure the water density variations. Figures 3 and 4 depict the first sector of the flume (from the station +0 to the station +200) and the cross section *A–A* (station +100), where many of the experiments were carried out. These figures indicate as well the arbitrary origin of the channel in the *x*, *y* and *z* directions.

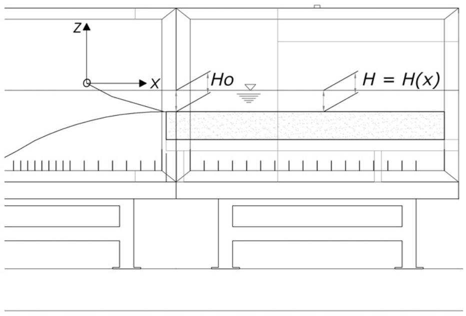

Fig. 3. The sector 2 of the flume

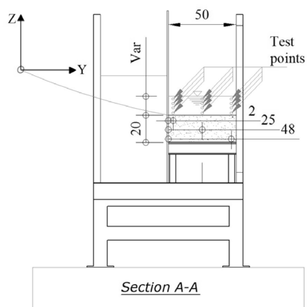

Fig. 4. The cross section *A–A* of the flume

Three different flow rates were used: 20, 30 and 40 dm^3/s. The tests were performed for three different cross sections (station +100 with the section *A–A*, station +300, and station +500) in twelve different points per cross section (the first series of experiments), these points are shown in figure 4. The results of the first ten series of experiments are depicted in figures 5, 6, 7 AND 8 for some chosen profiles:

- Figure 5: with Q = 30 dm^3/s station + 100.
- Figure 6: with Q = 30 dm^3/s station + 300.
- Figure 7: with Q = 40 dm^3/s station + 100.
- Figure 8: comparison of the profiles with Y = 48.

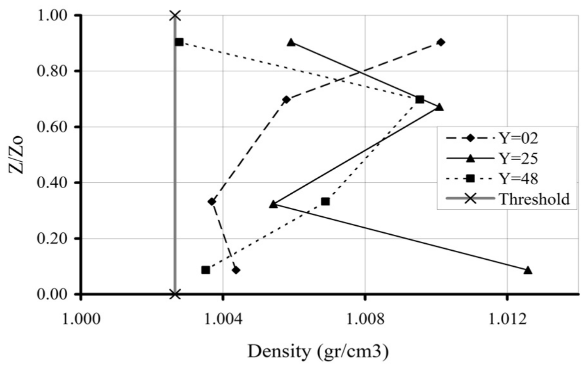

Fig. 5. Water density versus a normalized water depth

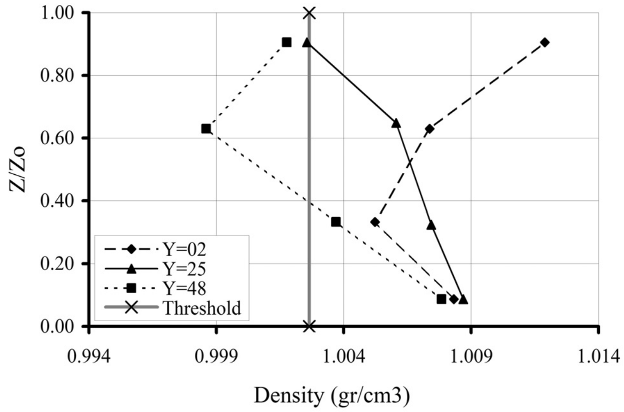

Fig. 6. Water density versus a normalized water depth

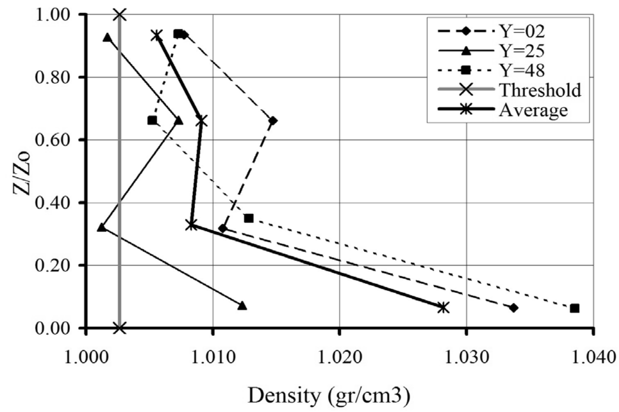

Fig. 7. Water density versus a normalized water depth

A normalized water depth was used (z/z_0), where z_0 is the total depth and z is the depth of the test, in order to have the same vertical scale for different flow rates and cross sections. Tests of the water were carried out with a siphon. All the samples were weighted and the volume of each of them was measured. Based on the definition of density, the experimental profiles were determined. In addition, clean water tests were carried out to establish a threshold value which allows us to compare the clean water density with the density of the mixture of suspended sediment and water. This threshold value is represented by the grey line that appears in every profile. The theoretical density at 19 °C is equivalent to 0.998 g/cm^3 (STRZELECKI and KOSTECKI [6]), while the average density of water from the clean water tests was 1.0026 g/cm^3. The density of the water tested at a flow rate of 40 dm^3/s (figure 7) is higher than the other experimental densities; this fact is evident, taking into account that the lifting forces are proportional to the average velocity (equation (4)). Hence, the suspended sediment concentration is higher, increasing the value of the density of the fluid in motion.

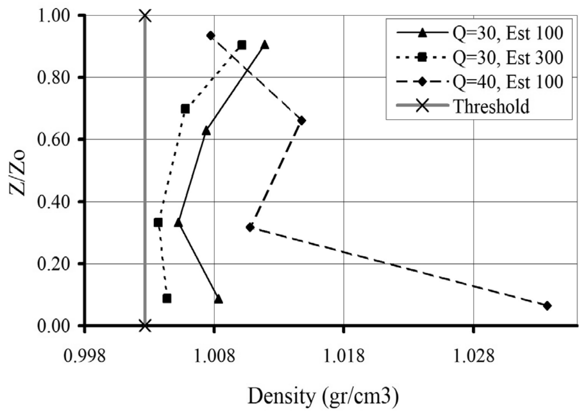

Fig. 8. Water density versus a normalized water depth

The density in figure 7 (assigned with the legend *average*) can be expressed by the function of the water depth z using the following formula:

$$\rho(z) = 1000(0.9482z^2 - 0.3176z + 1.0267)\,. \tag{5}$$

Expression (5) will be used as a definition for a model built in order to consider the water density as constant or non-constant which allows us to compare the results of both cases. Expression (5) is only valid for the established hydrodynamic conditions of the flume in a laboratory. The experimental results are sensitive, thus a new series of experiments are ongoing in the laboratory of WUT in order to refine the values of the experimental density profiles. Once it was proved that the density of the fluid varies, depending on the water depth, the kinematic viscosity varies as well and should not be considered fixed. This condition is fulfilled, when suspended load is transported by the fluid in turbulent motion.

The first assumption of Wang can be considered not valid (the interaction between flow and sediment movement can be neglected because the sediment concentration is low) since the concentration of the water is not low. The comparison of the results of the 2-D model, described in the fifth section, corroborates this fact.

4. THE FLOW THROUGH POROUS MEDIA

Figure 1 depicts the scheme of the water velocity profile along the z direction. This figure shows the case where the channel bed is made of porous medium, such as the movable sandy bed of the flume shown in figure 2. The velocity of the water at the boundary between the sandy bed and the water is considered to be zero in hydrodynamic modelling. A numerical model was built using the finite element program Flex PDE in order to estimate the value of the velocity at this boundary and to check whether or not this value can be neglected for modelling purposes. The Darcy's law is the governing equation of the model and the basis for modelling the seepage flow through porous media. Forchheimer proved that the hydraulic head is a harmonic function which fulfils the Laplace equation:

$$v_f = -k_f \cdot \operatorname{grad}\left(\frac{p}{\gamma} + z\right) = -k_f \cdot \operatorname{grad} H \,, \tag{6}$$

$$\nabla^2 \varphi = \nabla^2 H = \frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial z^2} = 0 \,, \tag{7}$$

where ϕ is the potential function of the flow represented in this case by the total hydraulic head. In the case of the open air flume, this hydraulic head varies linearly along the x–axis. Equation (1) was used to determine this hydraulic head and to establish the potential function. A very slow velocity field can be expected because the variation of the head is too small in the system. The permeability of the sand that forms the bed of the flume was estimated with the Allen Hazen formula (JUAREZ and RICO [1]). This formula establishes the relation between the D_{10} of the material and its permeability. Based on the sieve analysis of the sand that forms the bed of the flume, its permeability was estimated at approximately 0.017 cm/seg. Figures 9 and 10 depict

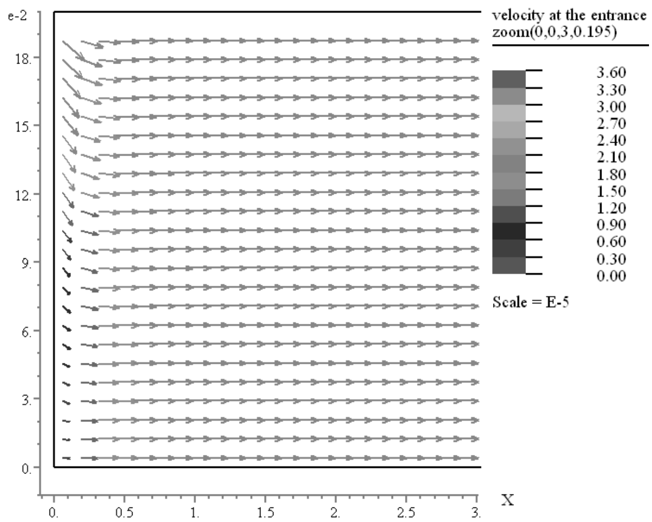

Fig. 9. The velocity field through the sandy layer

the modelled velocity field through the sandy layer for the flume shown in figure 2. The scale of the output demonstrates that this velocity in the whole layer and at its boundary with water can be neglected in hydrodynamic modelling. Nevertheless, the flow through the porous medium occurs. This can change in bigger channels, such as natural streams or rivers.

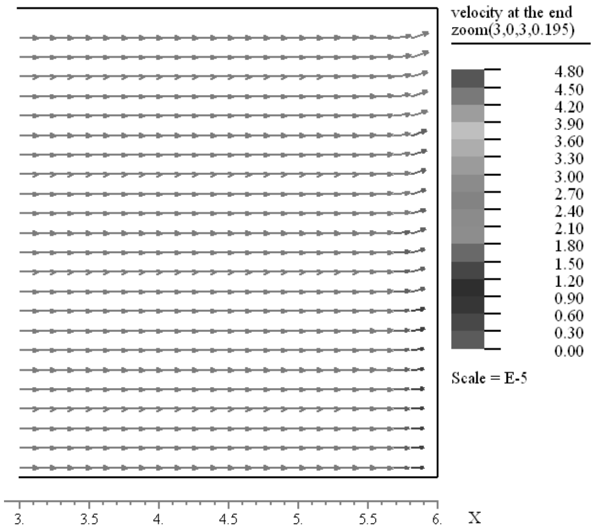

Fig. 10. The velocity field through the sandy layer

5. THE ROLE OF THE SANDY BED IN MODELLING FREE SURFACE FLOW

The second and third assumptions summarized by WANG and WU [7] demonstrate that the role of a movable bed in modelling open channel flow and sediment transport is neglected, especially if the material contains coarse or sandy soils. Hence, for the case of the flume in the laboratory, two models were built using these assumptions in order to check whether or not the sandy bed influences the results of the modelling and to discuss the previously mentioned assumptions:

- a uni-dimensional model based on HEC-RAS,
- a 2-D model using the Flex PDE program.

5.1. UNI-DIMENSIONAL MODEL FOR THE FLUME IN THE LABORATORY

The Hydrologic Engineering Centers-River Analysis System (HEC-RAS) is a commonly used uni-dimensional program, designed by the US Army Corps of engineers, which requires minimum data to calculate the energy of the system. Flow depths, water velocities, Froude number and shear stresses at the bed can be estimated. The modelled water depth and total energy of the system are shown in figure 11 at three different flow rates: 20, 30 and 40 dm^3/s. The modelled flow was calculated taking account of a fixed bed (the second assumption). Figures 12 and 13 compare the variation of velocities with time, calculated by HEC-RAS for 20 and 30 dm^3/s (represented by the lines marked with *HEC 20* and *HEC 30*, respectively) and the velocities measured in the laboratory (represented by *LAB 20* and *LAB 30*) in two cross sections (CS1 and CS2 in figure 11). Regardless of the steady-state flow, a fixed channel bed does influence the reliability of the model (see figures 12 and 13).

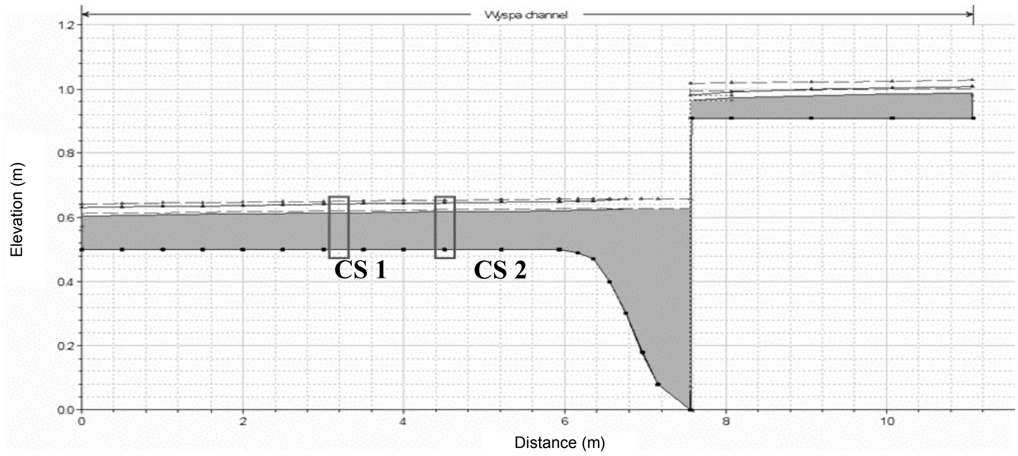

Fig. 11. Uni-dimensional model of the flume using HEC-RAS

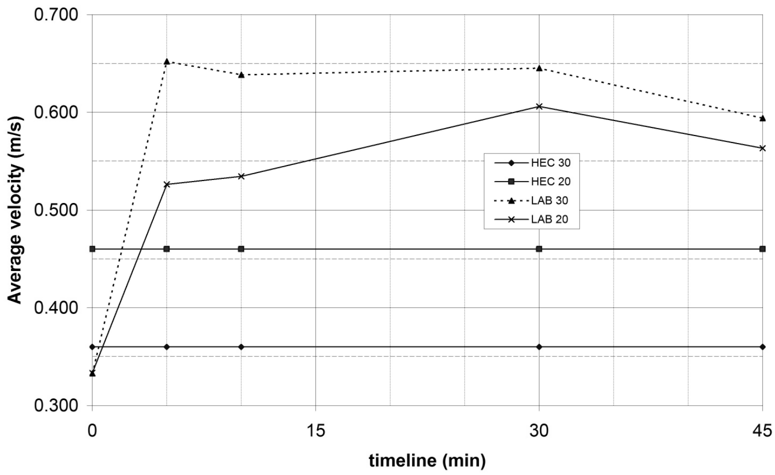

Fig. 12. Measured velocities at CS1 and CS2

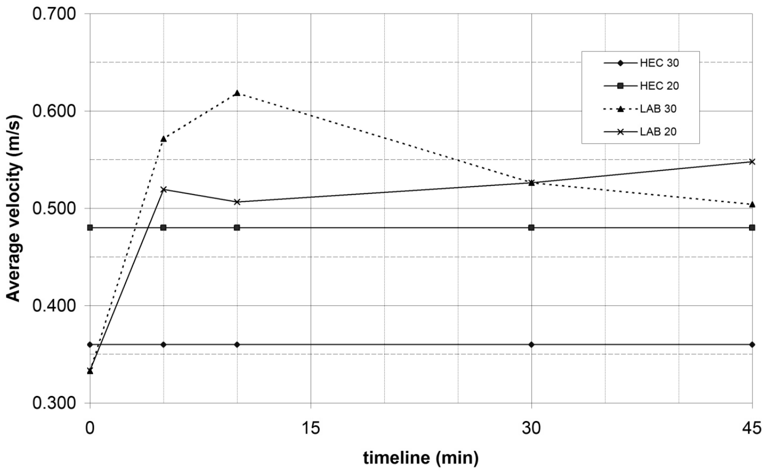

Fig. 13. Calculated velocities at CS1 and CS2

The third assumption (the interactions between different size classes of moving sediments are ignored) can influence the results because the roughness of the channel bed can change when sediment transport occurs. The roughness is another parameter that has to be introduced into many models. Figure 14 depicts the evolution of sieve curve representing the same material of the channel bed at the beginning of the experiment (the curve marked with *Test 1*) and at the end in three different stations (*Test I* at the station +100, *Test II* at the station + 300 and *Test III* at the station +500). If the sieve curve changes, the roughness of the bed material changes as well.

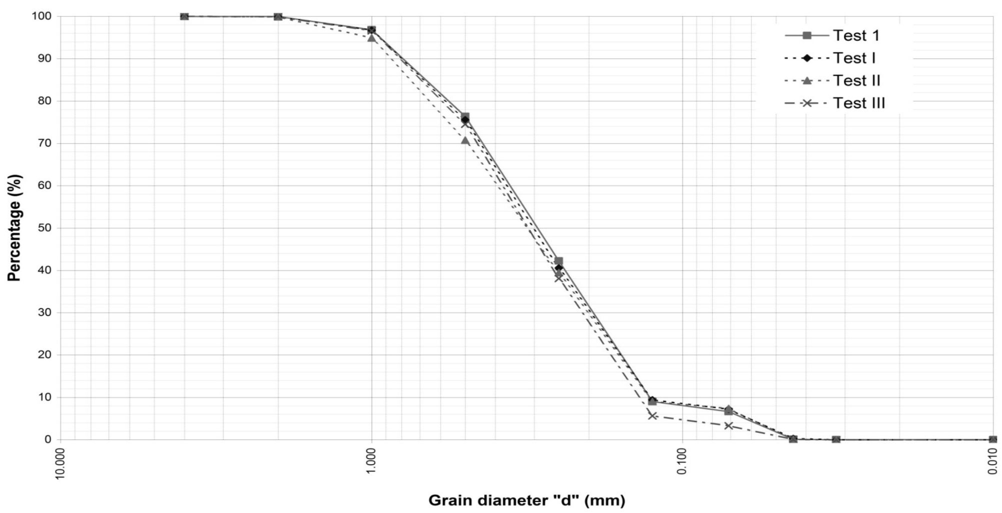

Fig. 14. Sieving curves of the bed material

5.2. TWO-DIMENSIONAL MODEL FOR THE FLUME

The first assumption of Wang is discussed because in our model, the sediment concentration in the fluid in motion is not low. Hence, a simple FEM-based model for laminar flow was built. It is necessary to stress that the model has to be upgraded because the majority of the channels, streams and rivers in the nature flow invariably in turbulent regime. Nevertheless, the model of a laminar flow allowed us to visualize the influence of the sediment concentration on the results. The governing equations of the model are the 2-D simplification of equation (2) for steady-state flow (no change of the velocity with time), represented by (8) and (9) and by the equation of continuity (10). Such a simplification assures the incompressibility of the fluid:

$$u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right), \tag{8}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\upsilon\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right), \tag{9}$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0\,. \tag{10}$$

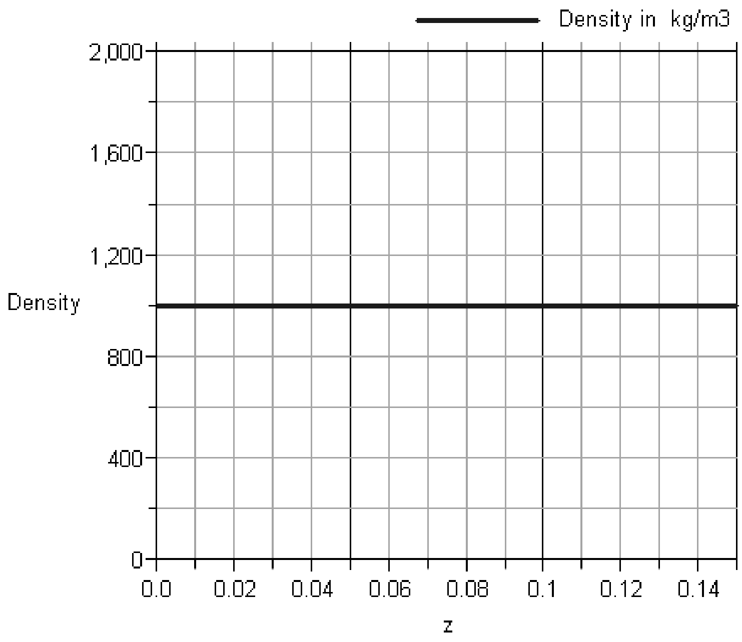

Fig. 15. Density distribution; density 1000 kg/m^3

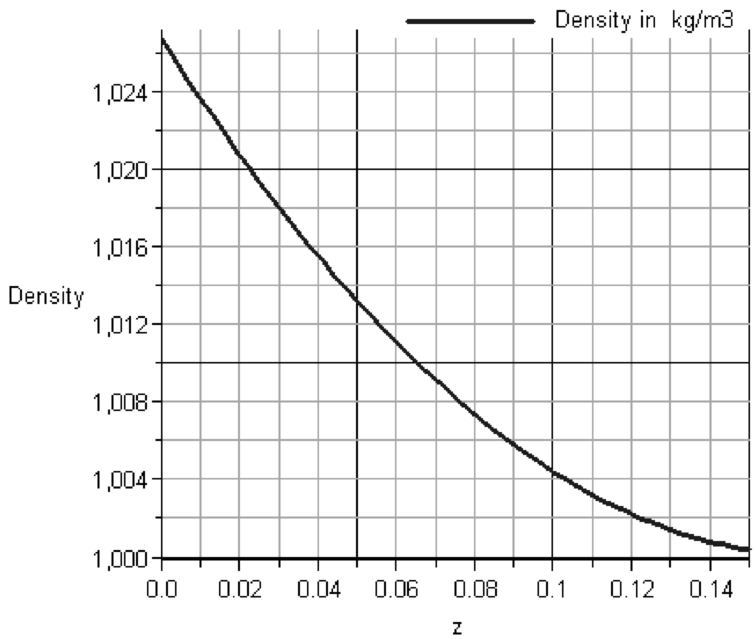

Fig. 16. Density distribution; density $f(z)$

Two trials were carried out for the geometry and flow conditions at 40 dm^3/s. In the first trial, the density is constant, and in the second one, the density versus the depth is represented by equation (5). Figures 15 and 16 depict these density distributions. To simplify the calculations, the viscosity is considered constant. The velocity in the z-direction is neglected in this model and the velocity in the direction of the flow (the x-direction) can be estimated using the law of the wall proposed by Prandtl:

$$\frac{u}{u_*} = \left(\frac{1}{\kappa}\right)\ln\left(\frac{z}{z_0}\right), \tag{11}$$

where u is the average downstream velocity, u_* is the bed shear stress, and κ is the Von Karman coefficient. Nevertheless, due to the complexity of the law of the wall, the velocity functions for 2-D model were not estimated using equation (11).

The following velocity functions were introduced as boundary conditions at the beginning of the channel (12) and at its end (13) based on the statistics and the average velocities measured in the laboratory:

$$u_1 = -56.134z^2 + 10.642z + 0.05\,, \tag{12}$$

$$u_2 = -61.747z^2 + 11.706z + 0.05\,. \tag{13}$$

Figures 17 and 18 depict the velocity fields at the beginning of the channel for both cases considered. Figures 19 and 20 depict the velocity distribution versus the water depth. When the water density is considered constant, the values of the modelled velocities are smaller than the values of the velocities estimated with variable density.

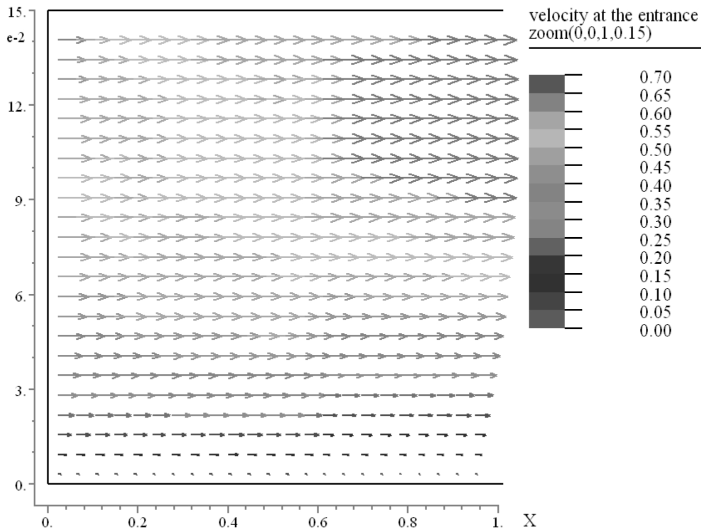

Fig. 17. Velocity field at the entrance to the flume (0–1 m) at constant density

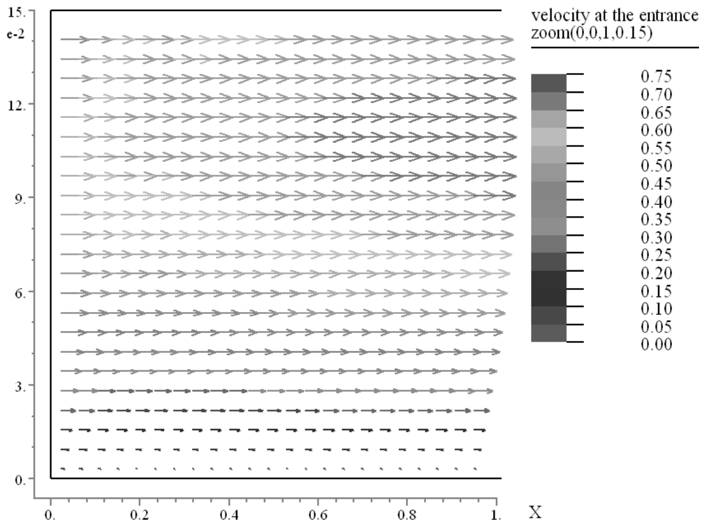

Fig. 18. Velocity field at the entrance to the flume (0–1 m) at variable density

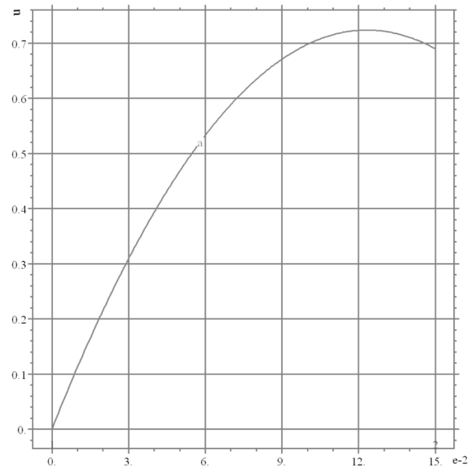

Fig. 19. Velocity in the middle of the flume at constant density

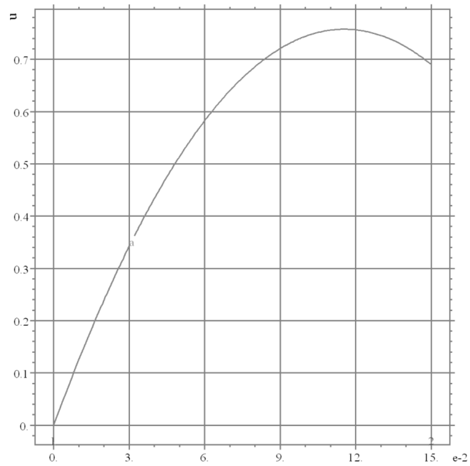

Fig. 20. Velocity in the middle of the flume at variable density

6. CONCLUSIONS

The following conclusions can be drawn:

1. A sandy bed does affect the hydrodynamic parameters that are considered always as constants for modelling open-channel flow and sediment transport, namely the molecular viscosity and the fluid density. The experiments in the laboratory of the WUT can represent the first approach to study in more detail this fact. These results are limited by certain hydrodynamic conditions and are to be upgraded.

2. At the boundary between the porous medium and the fluid, the latter moves with a certain velocity; nevertheless, this velocity can be neglected in hydrodynamic modelling.

3. A fixed bed affects the reliability of some 1-D models.

4. The roughness of the channel bed changes along the river bed due to sediment transport.

5. Based on the 2-D model it can be inferred that the omission of the interaction between flow and sediment movement does affect the results of the numerical modelling.

The 2-D model presented in this paper is valid for laminar flow, while in nature we mainly deal with turbulent flows. Hence, this model has to be upgraded. The model in which these conclusions will be taken into account is being built as the main topic of the author's PhD thesis.

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