Studia Geotechnica et Mechanica, Vol. XXX, No. 1–2, 2008

NON-LINEAR BIOT–DARCY'S PROCESS OF CONSOLIDATION

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Abstract: In this work, we present the problem of the creeping of the porous medium described by linear and nonlinear poroelasticity models based on physical relation in the Biot's body. The porosity function which depends on a dilatation of skeleton and fluid is introduced into the nonlinear model, and a process of medium deformation is also presented. The seepage in soil is calculated by the Finite Element Method for linear and nonlinear poroelasticity models, and the differences between these results are indicated. As a conclusion, some physical consequences of this type of nonlinearity in the Biot's model are discussed.

1. INTRODUCTION

A permanent development of numerical methods, equipment, and software enables us to resolve an engineering problems based on more complex mathematical models.

This allows us to obtain an improved approximation of the real physical occurrences.

Even simple rheology models are complicated in porous media mechanics. Thus, it is essential to search for a compromise between compounded mathematic description of the assumed model in microscopic and macroscopic scales and the ability to obtain a solution. A rheological model of porous medium, which is usually simplified to a model of ideally elastic body (Hooke's body), can serve as an example.

Laboratory research shows that the linear model of elasticity for porous medium is much too simplified, especially in scope pertaining to soil.

Physical compounds that define processes before reaching the boundary values are non-linear for soil medium. The framework of soil medium comprises separated grains or plates. The following forces may interact between them:

- the force of static friction between separate grains,
- the force of viscous friction, if a liquid phase (for example, in the case of coherent soil, water with electrostatic forces) occurs between grains,
- internal forces in the material cementing the grains of soil; such a material, having different mechanical characteristics due to chemical processes, can be obtained during seepage of mineralized water through the medium.

Deformation of medium due to load has a reversible character, when the movement is a result of elastic deformations of grains and or plates. In the case the movement occurs locally due to exceeding the limits of the forces mentioned above, the process of deformation has a non-reversible character.

Mathematical models describing the process of stress–strain, including the phenomena described above, would be for sure close to reality, but they would appear more complicated than the models usually applied to soil processes.

The purpose of this paper is to estimate the impact that the medium deformed by porosity in consolidation process has on the Biot's constants by using the numerical methods. The authors applied the convergence bi-scale method known from the theory of homogenization described by AURIAULT and SANCHEZ PALENCIA [1], AURIAULT [2], AURIAULT and the others [4]. The results of the numerical calculations of the deformation of medium porosity based on introduced hereinafter relations were compared with the results of deformation of the medium obtained on the basis of Biot–Darcy's classic model, where the constancy of the porosity is one of the basic assumptions.

2. PARAMETERS OF BIOT'S MODEL IN FUNCTION OF MEDIUM POROSITY

In order to define the relations between the parameters in Biot–Darcy's model and porosity, the two separate numerical problems have been solved in two-dimensional system:

(i) Flow of the Newtonian liquid through non-deformamable porous medium to define the relation between the Darcy's permeability tensor and the porosity of skeleton.

(ii) Deformation of the elastic porous medium to define the relation between the parameters in Biot–Darcy's model (that is the modulus N of shearing strain and the modulus A of bulky strain) and the medium porosity. The relations between Q and R, the other parameters of the model, and the porosity were based on FATT's and WILLIS' [7] as well as BIOT's and WILLIS' [5] researches.

A simple geometric shape of one periodic cell of square-shaped medium with circle inside that represents the grain of framework was used for the purpose of numerical analysis. The change of the medium porosity was obtained by changing the diameter of the circle.

It is obvious that in order to improve the analysis accuracy, a solid periodic cell model should be applied and the shape and the position of grains in the cell should be changed. However, the numerical analysis shows that a cell structure in micro-scale has only a small influence on the results. Therefore, the application of a simplified model is justified.

2.1. DEPENDENCE OF DARCY'S PERMEABILITY TENSOR ON POROSITY

The permeability tensor versus porosity for a two-dimensional constant flow through the periodic cell with the surface Ω, constrained by Γ, can be defined based on Stokes' equation and the continuity equation (AURIAULT [3], STRZELECKI and the others [8]):

$$-\frac{1}{\rho}\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial x^2}\right)=1, \quad x, y=\Omega_f,$$
$$-\frac{1}{\rho}\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial y^2}\right)=0, \tag{1}$$

$$\nabla\cdot\vec{v}=0, \quad \vec{v}=(u, v) \tag{2}$$

with the boundary conditions in the contact of the liquid with a solid body:

$$\vec{v}\big|_{\Gamma}=\vec{0}, \quad \frac{\partial p}{\partial n}\bigg|_{\Gamma}=0 \tag{3}$$

and with the periodic conditions on the boundary of a cell, where there is a liquid–liquid contact between neighbouring cells:

$$[u]=0, \quad [v]=0, \quad [p]=0. \tag{4}$$

The problem defined by equations (1)–(4) cannot be solved by a numerical method, because the system of equations is singular, therefore we suggest its modification by taking the divergence of equation (1):

$$\nabla\cdot(-1/\rho\cdot\nabla p+\mu\nabla^2\vec{v}+\vec{g})=0, \quad \vec{g}=(1, 0) \ \text{ in } \ \Omega_f. \tag{5}$$

Starting with a known mathematic identity in equation (5), the Laplace's operator should be replaced with $\nabla^2\vec{v}=\nabla(\nabla\cdot\vec{v})-\nabla\times\nabla\times\vec{v}$ and by use of (2) and the relation $\nabla\cdot(\nabla\times\nabla\times\vec{v})\equiv 0$ we arrive at:

$$\nabla^2 p=0. \tag{6}$$

Equation (6) is replaced with the flow continuity equation (2), and a new closing boundary condition is introduced as a result of homogenization theory (STRZELECKI and the others [8]):

$$\int_{\Omega_f} p\,d\Omega_f=0. \tag{7}$$

Problems (1)–(4) can now be resolved as follows:

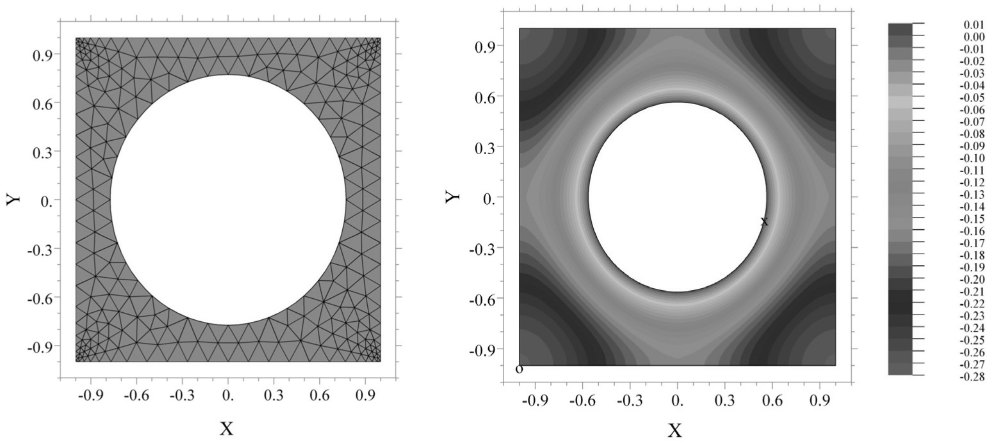
$$-\frac{1}{\rho}\frac{\partial p}{\partial y_1}+\mu\left(\frac{\partial^2 u}{\partial y_1^2}+\frac{\partial^2 u}{\partial y_2^2}\right)=1, \tag{8}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial y_2}+\mu\left(\frac{\partial^2 v}{\partial y_1^2}+\frac{\partial^2 v}{\partial y_2^2}\right)=0,$$
$$\nabla^2 p=0 \tag{8}$$

with the boundary conditions (3), the periodic condition (4) and the closing condition for a pressure function (7). A velocity field is the solution of this system. In accordance with the rules described by STRZELECKI and the others [8], the velocity field should be integrated by the area of the cell Ω_f to determine the permeability tensor components $\widetilde{k}_{11}$ and $\widetilde{k}_{12}$. In order to define other components, i.e., $\widetilde{k}_{21}$ and $\widetilde{k}_{22}$, we calculate the impact of an unit mass force along the y-axis, and then the procedure of the calculation of components is being repeated.

The square cell with the side length equal to 2 (in a non-dimensional system of coordinates) was applied in numerical tests based on the finite elements method.

Calculations for different grain positions in the cell have been made for this model. The results of the central position of the grain in the cell as well as finite elements mesh are presented in figure 1.



a) b)

Fig. 1. Calculation area – finite elements mesh (a), isoline horizontal velocity of filtration (b)

The permeability tensor was obtained in the course of the calculation. The tensor's components of $\widetilde{k}_{ij}$, except for the leading tensor's diagonal, can be assumed to be equal to zero. Because of the tensor's symmetry, the values on diagonal are equal, therefore, it can be assumed that the permeability tensor is defined by the permeability factor k.

Variation of the filtration factor k in the porosity function is presented in figure 2.

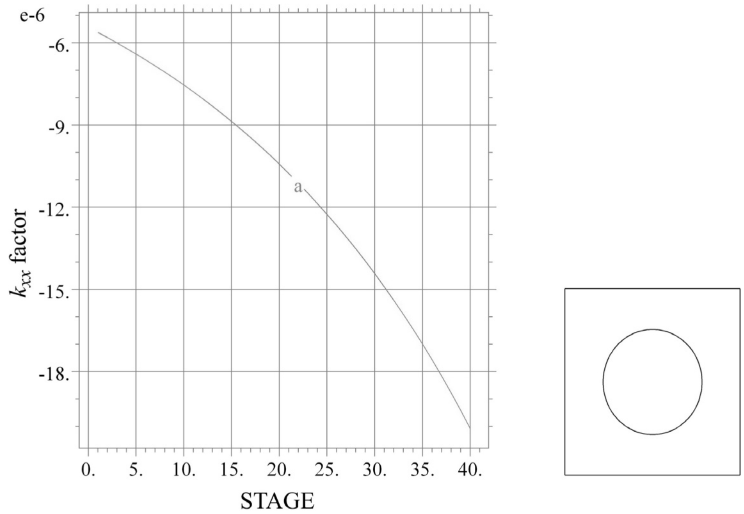


Fig. 2. Filtration factor versus porosity

2.2. RELATIONS OF *N* AND *A* (BIOT'S PARAMETERS) TO POROSITY

The boundary value problem of the deformation process of a solid phase of the medium, based on AURIAULT's studies [3], comprises only the solution of the following system of equations in micro-scale:

$$\frac{\partial}{\partial x_j}(a_{ijkh}e_{kh}(\vec{w})) = 0, \quad \vec{w} = (w_x, w_y), \quad x, y \in \Omega_s \tag{9}$$

with the boundary condition and the periodicity condition:

$$\begin{gathered} [(a_{ijkh}e_{kh}(\vec{w}))N_j]\big|_{\Gamma} = 0, \\ [\vec{w}] = 0, \quad [a_{ijkh}e_{kh}(\vec{w})] = 0, \end{gathered} \tag{10}$$

where $\vec{w}$ is the displacement vector, a_{ijkh} is the tensor of elasticity of order 4, e_{kh} is the deformation tensor of order 2.

Based on the procedures of bi-scale convergence we assume that all the functions under consideration depend on two variables defining macro- and microscopic changes accordingly. In the case of the two-dimensional problem, we consider the process of a periodic Ω_s cell deformation caused by a unit deformation from a macroscopic scale in a flat state of deformation. Physical relations of the flat model of elasticity can be expressed by the following equations in macroscopic scale:

$$
\begin{aligned}
\sigma_{xx} &= 2\widehat{N}E_x + \widehat{A}(E_x + E_y) + 2\widehat{N}\frac{\partial w_x}{\partial x} + \widehat{A}\left(\frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y}\right),\\
\sigma_{yy} &= 2\widehat{N}E_y + \widehat{A}(E_x + E_y) + 2\widehat{N}\frac{\partial w_y}{\partial y} + \widehat{A}\left(\frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y}\right),\\
\sigma_{xy} &= 2\widehat{N}\left[E_{xy} + \frac{1}{2}\left(\frac{\partial w_y}{\partial x} + \frac{\partial w_x}{\partial y}\right)\right],
\end{aligned}
\tag{11}
$$

where E_x – a relative extension along the x-axis, E_y – a relative extension along the y-axis, E_{xy} – a shearing strain.

The equilibrium system of equations has the following form:

$$
\begin{aligned}
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} &= 0,\\
\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} &= 0,
\end{aligned}
\tag{12}
$$

where $\widehat{N}, \widehat{A}$ are the Biot's constants in the case where the medium is non-porous, that means where it is a continuous body. The system (12) has to fulfil the boundary conditions and the periodic conditions with the closing integral condition:

$$
\int_{\Omega_s} w_x d\Omega_s = 0, \quad \int_{\Omega_s} w_y d\Omega_s = 0\,. \tag{13}
$$

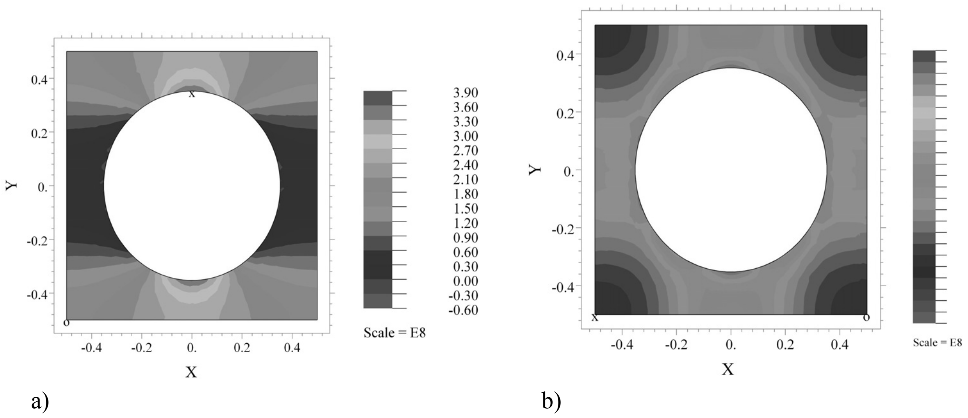


Fig. 3. Distribution of displacement vector in a local scale: vertical component of displacement (a), horizontal component of displacement (b)

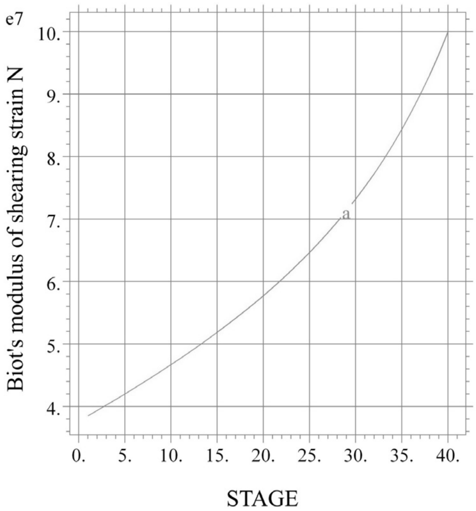
The calculations of the searched factor values are obtained by assuming $E_x = 1$, $E_y = 0$, $E_{xy} = 0$.

The analogical solution when $E_x = 0$, $E_y = 1$, $E_{xy} = 0$ can be obtained assuming the symmetry of an area geometry.

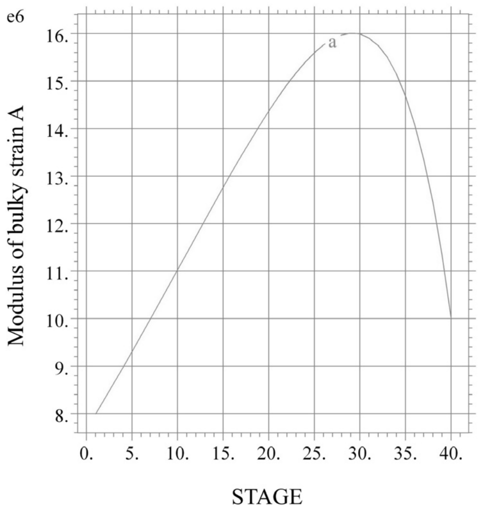
We arrive at the searched values $2N + A$ and A by determining of an average value of the components of w_x and w_y of the displacement vector.

The distribution of the displacement vector in the x- and y-axes' directions is presented in figure 3 in the scale of a periodic cell.

Figure 4 shows the function of the parameters N and M, depending on the medium porosity.



a)



b)

Fig. 4. Relations between Biot's constants and porosity: constant $2N$+A (a), constant A (b)

In order to identify the relations between other Biot's parameters and the medium porosity, more complicated system of equations in the microscopic scale ought to be solved.

However, provided that water is a liquid medium, we can assume that the fluid phase is less compressible than the skeleton of a porous medium.

The parameter R which defines the relations between liquid dilatation and the fuzzy pressure of liquid is linearly related to porosity.

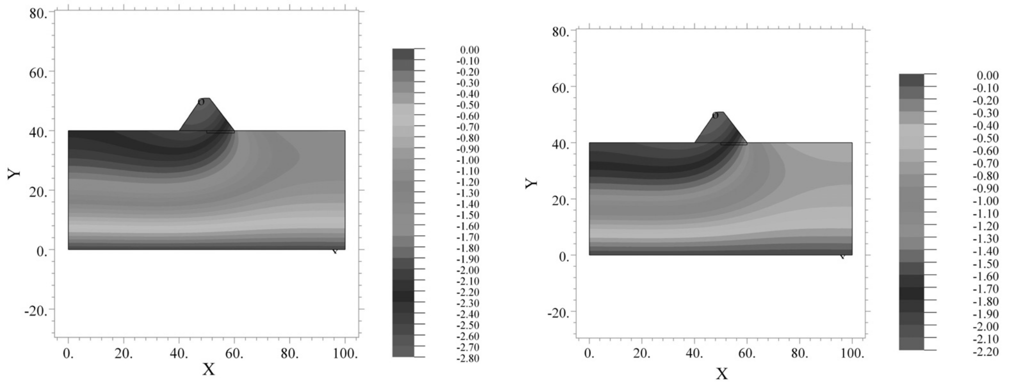
With reference to the studies of BIOT's and WILLIS' [5] and EMMERICH's [6] it can be stated that the following simple relations occur between Biot's parameters H and R and the porosity f:

$$H = R / f. \tag{14}$$

3. IMPACT OF NON-LINEAR MODEL ON THE SOLUTION OF CREEPING PROCESS

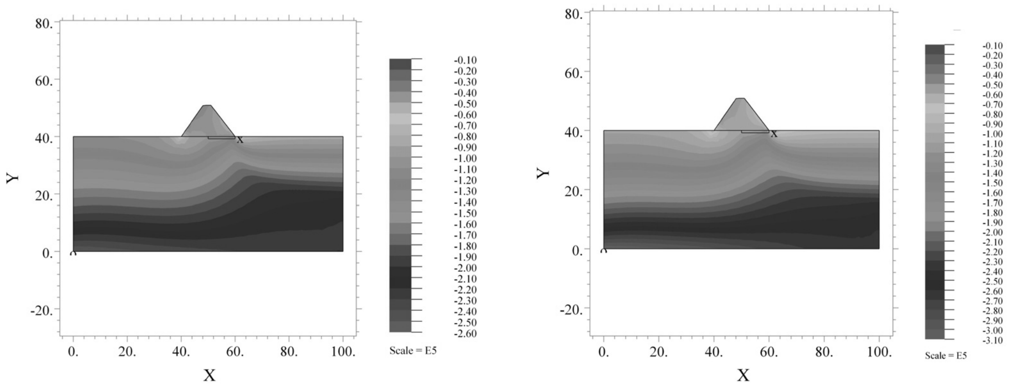
Two examples of dam and subsoil creeping under their dead-weight and water load accumulated in reservoir have been calculated in order to define the influence of non-linearity in the Biot's model on the solution. The first – based on the classical linear Biot's model, and the second – for the relations between Biot's parameters and porosity (non-linear model).

Calculations enable us to compare all important values: stress, strain and velocities of seepage. All the range of the differences between the linear and non-linear models cannot be discussed herein. Thus, only the comparison of the vertical strain and Coulumb–Mohr's plasticity potential are presented in figures 5 and 6 accordingly.



a) b)

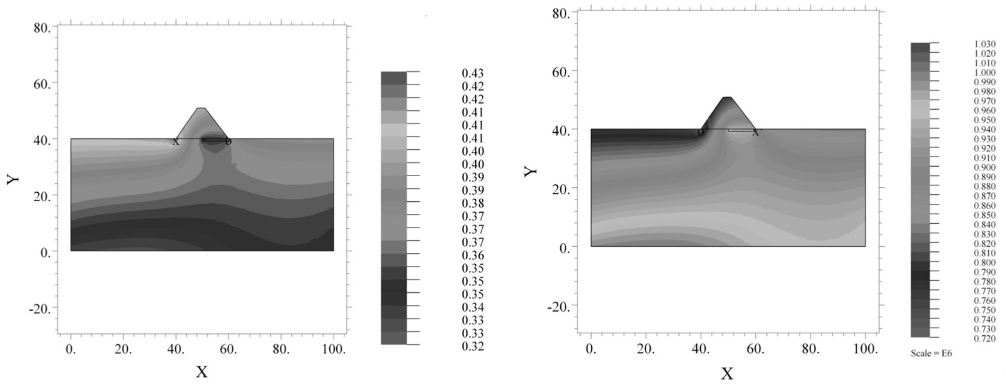
Fig. 5. Isolines of vertical displacement: linear model (a), non-linear model (b)



a) b)

Fig. 6. Isolines of Coulomb–Mohr's plasticity potential for: linear model (a), non-linear model (b)

As a result of deformation process, the initially homogeneous medium becomes nonhomogeneous and its effective factors are different from initial ones. Figure 7a presents the change of the medium porosity. The distribution of the value of N in the consolidation area is presented in figure 7b).



a) b)

Fig. 7. Isolines representing the porosity of consolidated medium after creeping process (a), isolines representing the value of the effective parameter N after consolidation (b)

4. CONCLUSIONS

Based on the foregoing numerical experiments it can be inferred that despite the fact that the model is not linear, the results of calculations are consistent with engineer's intuition which is based on classical linear Biot's model. Only the physical values calculated in the soil subjected to creeping are different. The character of the curves obtained in the calculations is also the same, both in linear and non-linear models.

It is obvious that the differences in the values of stress and strains would be more significant in the case of the stratified medium of soil.

This paper is one of the stages of a complex analysis of nonlinear seepage and consolidation.

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