Studia Geotechnica et Mechanica, Vol. XXXII, No. 2, 2010

CALCULATION OF A SHEET PILE WALL RELIABILITY INDEX IN ULTIMATE AND SERVICEABILITY LIMIT STATES

JERZY BAUER

Institute of Mining, Wrocław University of Technology,
Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland.

WOJCIECH PUŁA, MAREK WYJADŁOWSKI

Institute of Geotechnics and Hydrotechnics, Wrocław University of Technology,
Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland.

Abstract: The paper presents the procedure of calculating reliability index for a sheet pile wall loaded with a horizontal force. The procedure is carried out with the equations representing the ultimate and serviceability limit states. The calculation of the reliability index related to bearing capacity limit state was based on a response surface made of the largest bending moments in a sheet pile wall, while the reliability index of serviceability limit state was obtained by forming another response surface involving the maximum horizontal displacements of pile heads. The global reliability index was calculated with the use of the appropriate equations derived from system reliability analysis.

1. PRELIMINARY ASSUMPTIONS

Limit state design, which is in common use nowadays, involves the calculation of limit states of bearing capacity and serviceability. These calculations may be combined with reliability analysis resulting in probabilistic safety measures such as the probability of failure or the equivalent reliability index [1]. This kind of approach will be presented in this paper.

The calculations are made for a wall of reinforced concrete piles arranged in the form of a single row.

It has been decided that of all possible kinds of bearing capacity limit states, only the greatest bending moment, which may appear along the whole pile from its head to the base, will be analysed. The maximum bending moment cannot exceed the limit value M_{ult} resulting from the adopted reinforcement degree. Exceeding the limit value M_{ult} is considered a failure state. As regards the serviceability limit states, the horizontal displacement of pile head has been chosen from various kinds of these states. According to serviceability limit state principles, such a displacement cannot exceed a defined allowable value u_{all}. As in the previous case, exceeding the value u_{all} is considered to be a failure state.

As a result of random variability of soil properties, these conditions assume the form of two random events:

$$\{M \le M_{\mathrm{ult}}\}, \quad \{u \le u_{\mathrm{all}}\}, \tag{1}$$

where M stands for the bending moment at any cross-section of a pile, and u is a horizontal displacement of pile head. Failure-free state means that conditions (1) must be fulfilled. Taking account of the reliability analysis, it is then a two-component serial system and the problem consists in estimating the probability of failure:

$$P[(M \le M_{\mathrm{ult}}) \cap (u \le u_{\mathrm{all}})]. \tag{2}$$

In a general case, the resulting failure probability

$$p_F = 1 - P[(M \le M_{\mathrm{ult}}) \cap (u \le u_{\mathrm{all}})] = P\{(M > M_{\mathrm{ult}}) \cup (u > u_{\mathrm{all}})\} \tag{3}$$

is higher than the probabilities of failures obtained in the case where the conditions of bearing capacity and serviceability limit states are not met separately.

2. NUMERICAL ANALYSIS OF THE WORK OF A HORIZONTALLY-LOADED SHEET PILE WALL

The aim of a numerical analysis based on the computational model shown in figure 1 was to obtain a set of maximum bending moments in the sheet pile wall and of pile head horizontal displacements for the given values E_1 and E_2 of Young's modulus in the upper and middle layers of soil, respectively. These values will make it possible to find the response surfaces for the bending moment and for displacement. The computational FEM model presented in figure 2 incorporates three layers of soil.

In the first two layers, 4 m deep each, the elastic moduli E_1 and E_2 are treated as random variables. The third layer of a good quality ground is assumed to be homogeneous with non-random elastic modulus E_3 (see figure 1). The pile sheet wall with the mean thickness of 0.44 m reaches the depth of 9 m and is transversely loaded with the force P = 44 kN/m, applied to the pile head. In figure 2, two vertical lines near the wall limit the area with a higher number of triangular six-node finite elements.

Two assumptions have been imposed on the numerical model. The first is that the first layer is completely saturated, while the second assumption is that the loading is repeatedly applied and acts for a short period of time. The assumption that the loading acts in an almost pulsing way in saturated soil makes it possible to assume full contact between the pile and the surrounding soil on the left-hand side of the wall where the tensile stresses appear. The calculations confirm that the value of these tensile stresses do not exceed 0.025 MPa, i.e. 1/4 atmospheric pressure. If the loading in saturated soil changes quickly, the pile movement causes suction to act on the left-hand side of the wall. This justifies the assumption of a complete connection between the soil and the sheet pile wall.

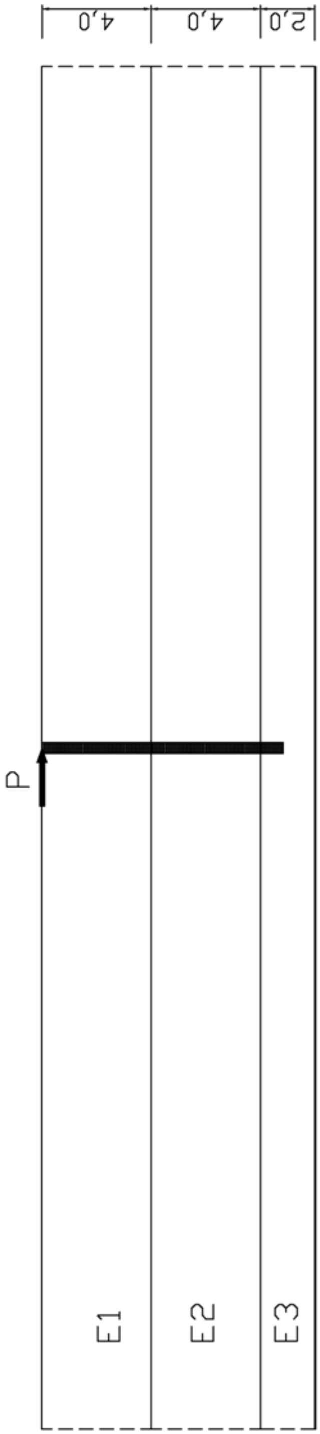

Fig. 1. Sheet pile wall embedded in a layered soil and subjected to horizontal force

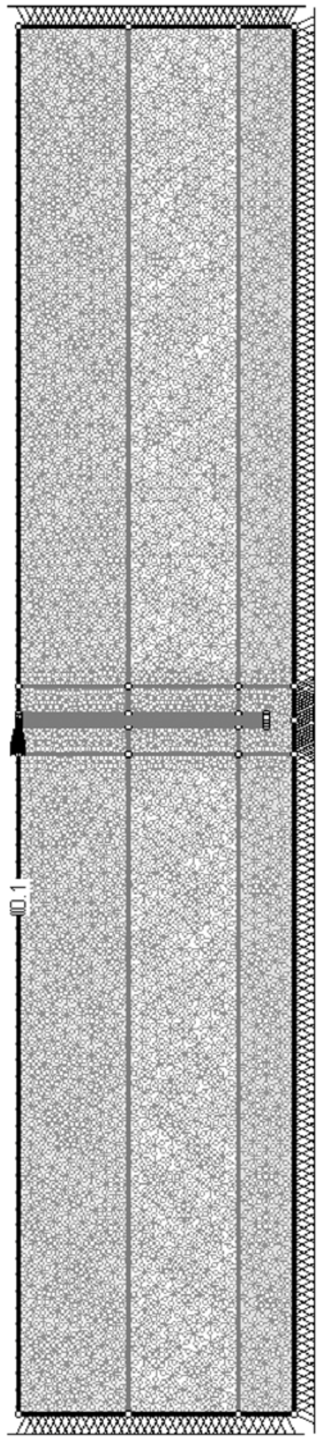

Fig. 2. A computational finite element model

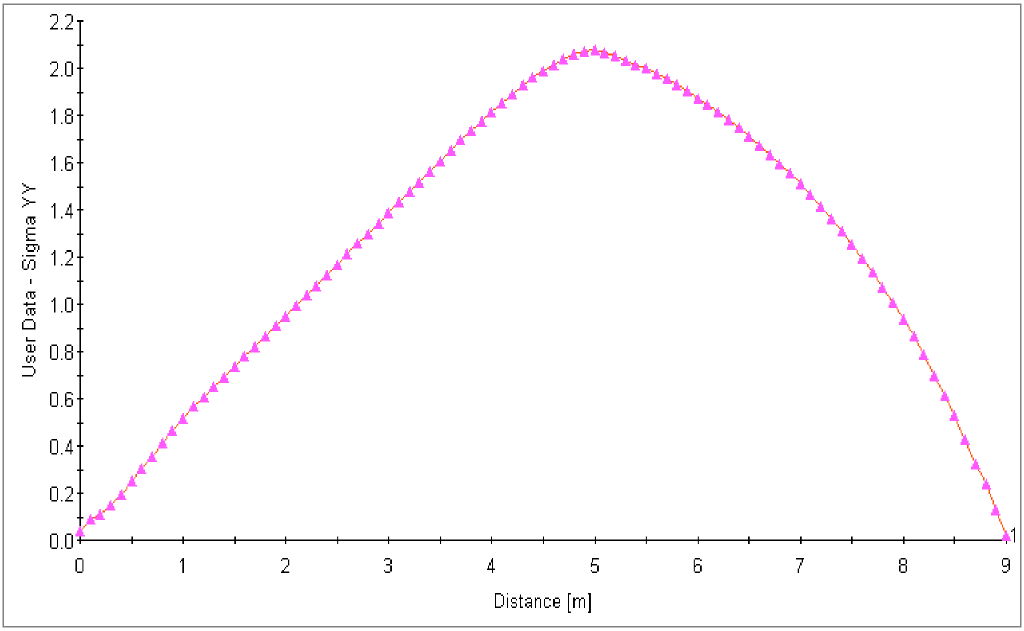

Fig. 3. Compressive stresses in extreme pile fibres

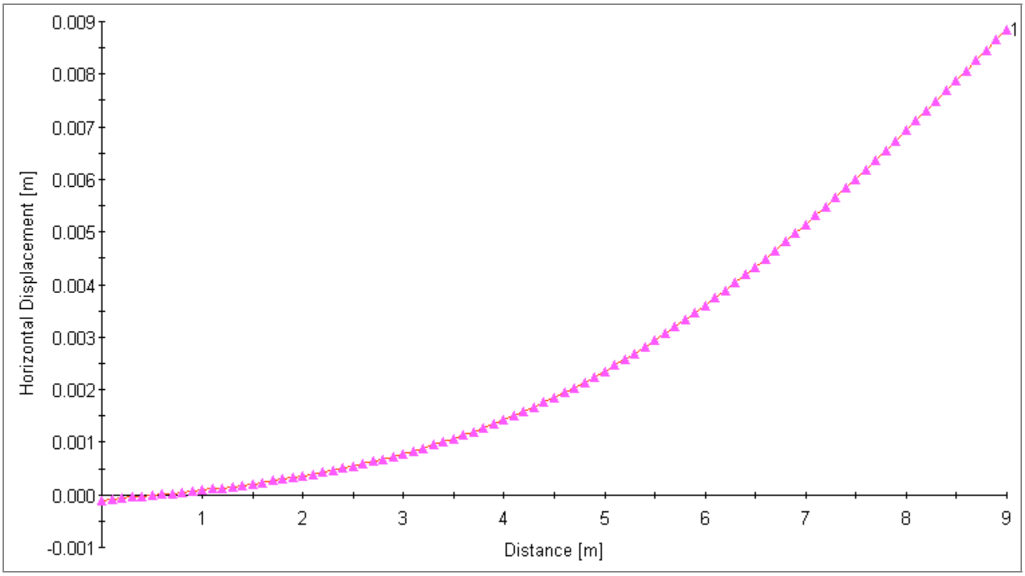

Fig. 4. Horizontal displacement stresses along sheet pile wall

The curve in figure 3 presents the compressive stress σ_y in the extreme fibers of the piles. Relation

$$M = W\sigma_y \tag{4}$$

enables the calculation of bending moments, where W is the bending index corresponding to one linear metre of the wall: $W = 1\cdot(0.44)^2/6$. Figure 4 presents the horizontal displacements of the wall. The maximum displacement corresponds to the displacement at the pile head.

The characteristics of random variables corresponding to material properties used in the numerical computations are gathered in table 1.

Table 1

Expected values of elastic constants and standard deviations of subsoil computational model

Layer types	Mean values of Young's modulus E (MPa)	Values of Poisson's ratio ν	Standard deviations of Young's modulus σ_E (MPa)	Probability distributions of Young's modulus E
Upper layer	4	0.30	0.45	lognormal
Middle layer	20	0.30	3.00	lognormal
Bottom layer	80	0.35	0	deterministic
Concrete sheet pile wall	28 000	0.16	0	deterministic

The computations were carried out with the use of the finite element software PHSESES2, specially designed for analysing the stresses and displacements of underground structures in plain strain problems.

3. ULTIMATE AND SERVICEABILITY LIMIT STATES OF A PILE SHEET WALL

According to Polish Standard [2] the correctness of the structure designed should be checked through analysing the limit states of bearing capacity and serviceability. As the pile sheet wall will be subjected to bending, the requirement of not exceeding the bearing capacity in the most loaded cross-section implies the first inequality in (1). The values of the ultimate bending moment M_{ult} depend on the percentage of reinforcement in a particular cross-section (table 2). These values are determined with a simplified method that is based on normative correlations for the cross-section height h = 0.44 m, the concrete cover thickness a = 0.05 m and the design plasticity limit of steel plasticity f_s = 210 MPa.

Table 2

Values of boundary moment depending on reinforcement percentage

Reinforcement percentage (%)	A_s (cm^2/m)	A_{s1} (cm^2/m)	M_{ult} (kNm/m)
0.50	22.00	11.00	78.54
0.55	24.20	12.10	86.40
0.60	26.40	13.20	94.25
0.65	28.60	14.30	102.10

A_s – the reinforcement surface, A_{s1} – the surface of tensioned reinforcement, M_{ult} – the maximum value of bending moment.

The condition of not exceeding the boundary value of the horizontal displacement $u_{\text{all}} = 1.1$ cm by pile heads produces the second inequality in formula (1).

In the next section of this paper, it will be assumed that the bending moment M and the horizontal displacement u are the functions of two random values of the elastic modulus E and the random horizontal force P.

4. RESPONSE SURFACES FOR BENDING MOMENT AND DISPLACEMENT

In order to obtain the reliability measures, the functional relationships between ‘input’ random variables (the values of elastic modulus) and those obtained ‘at the output’ of the problem (bending moment, head displacement) should be known. Unfortunately, no explicit functional relationships between these quantities are known and it is only possible to define, by means of finite element method, the set of relevant physical quantities for the adopted values of material constants. In this situation, the response surface method was used to calculate the reliability indices. The method generally consists in approximating an unknown function, for which only a certain amount of values is known, with an adequately adopted function. The choice of approximating function may be based on the results of experimental research as well as on the results of numerical calculations, e.g. the results obtained by finite element method. A detailed information can be found in numerous monographs, e.g. that by MYERS and MONTGOMERY [3].

This paper uses an iterative algorithm based on nonlinear regression and the Marquardt method, adapted by BAUER and PUŁA [4]. In the case discussed, the response surface has the form of a second degree polynomial in two variables, i.e.:

$$f(E_1, E_2) = \frac{P}{44}(b_1 + b_2E_1 + b_3E_2 + b_4E_1^2 + b_5E_2^2 + b_6E_1E_2) + \varepsilon\,, \tag{5}$$

where ε is the random variable defining a fitting error (cf. BAUER and PUŁA [4]), E_1, E_2 are random values of Young’s modulus and b_1–b_6 – the coefficients of the desired response surfaces *f*. The function *f* was determined fivefold – once in the case of approximating the displacement u and four times – for approximating the maximum bending moment M, depending on the boundary value (c.f. table 2). The values B_1–B_6 of coefficients b_1–b_6 obtained in particular cases are presented in table 3.

Table 3

Coefficients defining particular response surfaces

Coefficients of response surfaces	u =1.1 (cm)	M_{bound} = 78.54 (kNm/m)	M_{bound} = 86.40 (kNm/m)	M_{bound} = 94.25 (kNm/m)	M_{bound} = 102.1 (kNm/m)
B_1	2.556	115.77	132.851	136.391	148.8
B_2	–0.405	–23.7828	–32.5212	–33.3744	–37.27
B_3	–0.0472	1.61518	1.33108	1.13407	0.6371
B_4	0.0273	1.75618	3.03030	3.11103	3.722
B_5	0.616E-3	–0.02001	–0.0148148	–0.0116667	–0.4866E-2
B_6	0.324E-2	–0.022728	–0.674 E-9	0.016661	0.04807

5. COMPUTING RELIABILITY MEASURES

Reliability is calculated in two steps. The first step is to calculate the probabilities of failure

$$p_{F_1} = P\{M > M_{\text{ult}}\} \tag{6}$$

connected successively with four values M_{ult} of boundary bending moment and the reliability indices β_1 related to these probabilities by the following equation

$$p_F = \Phi_0(-\beta)\,, \tag{7}$$

where Φ_0 is the cumulative normal distribution function.

The second step is to calculate the probability

$$p_{F_2} = P\{u > u_{\text{all}}\} \tag{8}$$

connected with exceeding allowable horizontal displacements and the reliability index β_2 related to this probability (equation (7)). These measures were calculated by means of the SORM method, widely used in the reliability theory (cf. HOHENBICHLER et al. [5] or PUŁA [6]). The reliability indices β_1 corresponding to the probability (6) of not meeting the condition of ultimate limit state and the reliability indices β_2 corresponding to probability (8) of not meeting the condition of serviceability limit state can be found in table 4.

Table 4

Values of reliability indices depending on reinforcement percentage

Reinforcement percentage (%)	β_1	β_2	β
0.50	1.41	2.31	1.38
0.55	2.90	2.31	2.27
0.60	4.37	2.31	2.31
0.65	5.83	2.31	2.31

However, as mentioned in section 1, our aim was to determine the probability of failure:

$$p_F = P\{(M > M_{\text{ult}}) \cup (u > u_{\text{all}})\}\,, \tag{9}$$

and the global reliability index β related to this probability through equation (7). The method for determining the global reliability index, one of the problems included in system reliability analysis, is very briefly described below.

In practice, in order to fulfil the safety conditions, a structure must often meet several various, often interrelated, criteria. Therefore one should adopt a model whose structure is the system of components, each of them satisfying different failure criteria characterised by different limit state functions. These elements can form a series system (like in the example discussed in this paper) – then the failure-free operation of the whole system requires failure-free operation of all the elements. The opposite of the system of components is a parallel system, where a lack of failure in one element is sufficient for the failure-free operation of the whole system. A mixed system is a combination of the parallel and serial systems. In calculating the probability of such a system it is usually necessary to estimate the probability of the following event:

$$F_{\text{sys}} = \bigcup_i \bigcap_j \{g_{ij}(\mathbf{X}) \le 0\}\,, \tag{10}$$

where g_{ij} stands for the limit state function of the element ij. Therefore it is necessary to estimate the probability of the union and intersection of events. If the events analysed are not mutually exclusive, the so-called Ditlevsen's bounds are often applied (DITLEVSEN [7]):

$$P\left(\bigcup_{i=1}^{q} F_i\right) = \begin{cases} \le \sum_{i=1}^{q} P(F_i) - \sum_{i=2}^{q} \max_{j<1}\{P(F_i \cap F_j)\} \le \sum_{i=1}^{q} P(F_i), \\ \ge P(F_1) + \sum_{i=2}^{q}\left[\max\left\{0, P(F_i) - \sum_{j=1}^{i-1}\{P(F_i \cap F_j)\}\right\}\right] \ge \max_{i=1}^{q}\{P(F_i)\} \ge 0. \end{cases} \tag{11}$$

If the sum on the right-hand side of inequality (11) is higher than unity, the upper limit is, of course, unity. As we can see, the use of inequality (11) requires calculating the probabilities of intersections in the form of $P(F_i \cap F_j)$. These probabilities can be estimated after employing transformation for standard normal space. It can be proved that for two-dimensional cumulative normal distribution function Φ_2 with mean (0, 0) and variance (1, 1) and correlation coefficient between the components ρ, the following equality is valid:

$$\Phi_2(x, y, \rho) = \Phi_0(x)\Phi_0(y) + \int_0^{\rho} \phi_2(x, y, t)dt, \tag{12}$$

where ϕ_2 stands for the density corresponding to Φ_2 (cf. [1]). This equality leads to the following approximate formula:

$$P(F_i \cap F_j) \approx \Phi_0(-\beta_i)\Phi_0(-\beta_j) + \int_0^{\rho_{ij}} \phi_2(-\beta_i, -\beta_j, t)dt, \tag{13}$$

where ρ_{ij} stands for the coefficient of correlation between the random variables $g_i(\mathbf{X})$ and $g_j(\mathbf{X})$, while g_i, g_j are limit state functions for the elements i and j, respectively, and β_i and β_j – the corresponding reliability indices according to FORM method.

When applying the above-mentioned Ditlevsen's bounds, the probability (9) corresponding to four cases of reinforcement surfaces was calculated as well as the global reliability indices β were listed in table 4.

6. FINAL REMARKS

When analysing the computation results of the reliability given found in table 4, one can describe three different situations. The reliability indices in the first row of table 4 refer to the case of a relatively low value of boundary moment resulting from a small percentage of reinforcement. The possibility of the system failure is connected mostly with the probability of exceeding the condition of ultimate limit state with the reliability index β_1 = 1.41. This probability of failure is slightly increased towards the global probability of failure due to additional possibility of exceeding the condition of serviceability limit state with the reliability index β_2 = 2.31, which produces the value β = 1.38 of the global reliability index. In the case presented in the second row of table 4, the situation is opposite. The system failure chiefly does not meet the condition of serviceability limit state. The increased value of boundary moment reduces the probability of failure resulting from not meeting the condition of bearing capacity limit state, which is reflected in the increase of reliability index to the value β_1 = 2.90, and only slightly reduces the value of global reliability index – from the value β_2 = 2.31 related to exceeding the condition of serviceability limit state to the global value β = 2.27.

The third case, shown in rows 3 and 4 of table 4, is different. System failure is chiefly caused by exceeding the condition of serviceability limit state, and the global reliability index β has the same value as the coefficient $\beta_2 = 2.31$ related to serviceability limit state. This means that there is practically no stochastic interaction between the conditions of ultimate limit states and serviceability limit states. This situation results from the fact that the failure area related to exceeding boundary moment lies far from the origin of the coordinate system of standardized random variables of the material constants E_1 and E_2. One should note that the third case is the most advantageous if a structural safety is taken into account.

ACKNOWLEDGEMENT

This article, as a research project, was supported by the funds for science in 2008–2010.

REFERENCES

[1] DITLEVSEN O., MADSEN H.O., *Structural Reliability Methods*, John Wiley & Sons, Chichester, 1996.
[2] Polish Standard. PN-B-03264, *Concrete, reinforced-concrete and compressed structures. Static calculations and design* (in Polish), Warszawa, 2002.
[3] MYERS R.H., MONTGOMERY D.C., *Response Surface Methodology Process and Product Optimisation Using Design Experiments*, John Wiley and Sons, New York, 1995.
[4] BAUER J., PUŁA W., *Reliability with respect to settlement limit-states of shallow foundation on linearly-deformable subsoil*, Computers & Geotechnics, 2000a, 25, No. 3–4, 281–308.
[5] HOHENBICHLER M., GOLLWITZER S., KRUSE W., RACKWITZ R., *New light on first- and second-order reliability methods*, Structural Safety, 1987, 4, 267–284.
[6] PUŁA W., *Applications of structural reliability theory in the assessment of foundation safety* (in Polish), Wrocław, Oficyna Wydawnicza PWroc., 2004.
[7] DITLEVSEN O., *Narrow reliability bounds for structural systems*, Journal of Structural Mechanics, 1979, Vol. 7, No. 4, 453–472.