Studia Geotechnica et Mechanica, Vol. XXXII, No. 3, 2010

KINEMATIC METHOD FOR DETERMINING INFLUENCE FUNCTION OF INTERNAL FORCES IN THE STEEL SHELL OF SOIL-STEEL BRIDGE

CZESŁAW MACHELSKI

Institute of Civil Engineering, Wrocław University of Technology,
Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland.

Abstract: The live loads of soil-steel bridges are vehicles, and their influence on the steel shell is exerted through the roadway surface. To obtain the maximum values of internal forces in the element analyzed it is essential to place the vehicles in an appropriate manner over the structure. The influence surface of the internal forces in the shell can be utilized in this task, with the reference to the level of roadway. In the paper, there are presented the methods which enable the influence functions of internal forces to be obtained by means of kinematic constraint concept. It is based on the imposition of the basic deformation which is suitable for the internal force in the considered element of the system. Further, on the basis of the deformations inside the element the boundary forces are calculated. They are substitute load of nodes of this element in the global model of structure, which represents the influence function of internal force. Practical calculations of influence function are carried out by means of FEM, using the substitute load of nodes of the element. Due to this load, there is created the surface of the vertical displacements of a roadway (corresponding to the direction of external loads) which constitutes also the influence function of internal forces. The algorithm proposed can be applied in any geometrical model, provided that the analyzed element in which the force is to be found is the bar. The model does not require any change of discretization when choosing next element or internal force for the analysis. Simple formulas presented allow the boundary forces to be calculated. They can be used to determine the influence functions of internal forces and normal stresses in the soil-steel bridges.

1. STRUCTURAL ELEMENTS OF SOIL-STEEL BRIDGE

In the steel-soil structures shown in figure 1, the structural elements are as follows: steel corrugated plate, backfilling surrounding it and interacting with it and roadway surface. The main element in this system is the soil backfilling. Its influence on the behaviour of the structure has been analysed in numerous papers [1]. Among the essential factors which influence the physical and strength parameters of backfilling are usually:

- the type of soil, its composition and granulation;
- the degree of soil compacting during construction period and the equipment used during this process;
- the service period of the structure;
- the height of the backfilling above the top of the shell;
- the type of the roadway surface and the type of substructure.

The examples of the results of the research conducted by PETTERSSON [2], which show the influence of the two last factors, are given below. The structure under consideration was made from the steel plate MP 200 × 55 × 3 (the dimensions $a \times f \times t$ stand respectively for: the wave length, the wave height and the plate thickness, see figure 4b). A pear-shaped cross-section of the shell was closed and had the following dimensions: h = 4.55 m (height), L = 6.04 m (length), R = 3.052 m (the radius of the top curvature), R_n = 1.308 m (the radius of the shell in the corners). The loader was used in this research as the moving load with the following technical parameters: the load of the front axle P_1 = 221 kN (with a scoop full of soil) and rear axle P_2 = 69 kN; the track of wheel a_{12} = 3.40 m and the wheel base of both vehicle axles 2.20 m. During the site measurements, the load was moving in the quasi-static manner along the axis of roadway, prepared specially for this research.

Fig. 1. The view of soil-shell structure

Table 1 shows the results of the analysis giving the influence of the roadway surface shape and substructure on the bending moment in the circumferential strip at the top of the shell, at the point below the line of wheels of vehicle which crosses the bridge. The following variants of the shape of road surface were considered, provided that its overall thickness is constant and equal to 0.75 m:

- A4, gravelly sand only was used in test series A;
- B1, after approximately eight months (winter period) and a couple of hundred passes of heavy vehicles the degree of compaction was increased significantly;
- B2, 0.20-m top of gravelly sand was exchanged for base course gravel (for grading curve, with high internal friction);
- B3, the same as cover B2, a geogrid added between the two soil layers;
- B4, 0.05-m top of base course gravel exchanged fot asphaltic surface course.

Table 1

Results of measurements [2]

Test	A4	B1	B2	B3	B4
M (kNm/m)	6.93	5.96	4.80	4.80	5.66

The thickness H of the backfilling has a crucial influence on the values of internal forces and deformation of shell. This is exemplified by the research whose results were shown previously in [2]. In table 2, there are compared the results of measurements (deflections) and calculations obtained on the basis of unit deformations (internal forces). The values of the bending moment M and axial force N are shown with reference to the width of circumferential strip of the shell. In deflection values, there are discerned maximal value w and the value w_r (shown also in table 2) which remained after the site tests. In that case, the roadway was without road surface, denoted as A4 in table 1. The curves in figure 2 show that the changes of internal forces and deflections as the functions of the thickness of backfilling are similar. An increase in backfilling thickness over the shell has a significant influence on the reduction of internal forces and deflections.

Table 2

The results of site research

Cover heigh H (m)	Internal forces		Displacement (mm)		Factor $\frac{w}{w_r}$ (%)	Factor k (kN/mm)
	M (kNm/m)	N (kN/m)	w	w_r		
0.75	6.93	121.4	6.77	0.15	2.22	32.6
0.90	3.70	80.4	4.32	0.10	2.31	51.2
1.20	1.85	57.0	2.97	–0.08	–2.69	74.4
1.50	1.10	43.0	1.86	–0.13	–6.99	118.8

An appropriate measure of the flexibility of the soil-steel structures is the following factor

$$k = \frac{Q}{w} \tag{1}$$

which is calculated as the ratio of the loading of the structure Q to the displacement w [1] due to this load. The values of these factors, given in table 2 (when $Q = P_1 = 221$ kN), are smaller than these to be found in such type of structures which proves the great flexibility of the structure being examined.

Based on the parameters given before it can be concluded that the soil-shell structure under live load is the basic load carrying structural element of the bridge. In the soil-steel structure, the backfilling has two, apparently opposed tasks. From one point of view, it is the significant constant dead load, which must be carried by the shell at the stage of construction. From the other one, it creates the elastic surrounding of the shell, which can increase the load carrying capacity many times when subject to the live loads [1].

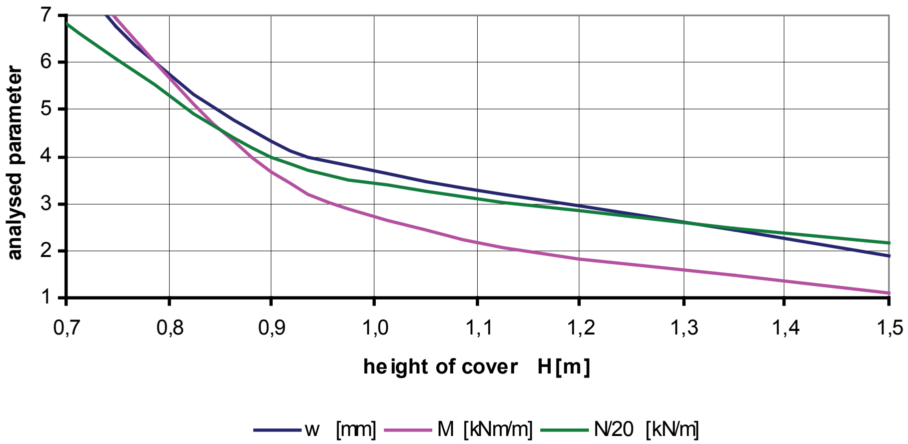

Fig. 2. Internal forces and deflections as the functions of the backfilling thickness *H*

Soil which surrounds the shell is designed in such a way as to obtain its required parameters as a structural material. In order to arrive at an effective interaction between soil and shell, it is necessary to utilize the aggregate of an appropriate quality and properly compacted layers of backfilling around the shell [1]. It is not desirable to use a cohesive soil. There are also encountered the cases of using the light aggregate as the backfilling. It is specially advantageous when the structures are constructed on the high embankments, being founded on a week soil (soil without the load carrying capacity). The choice of proper equipment for the soil to be used is very import in this case.

2. INFLUENCE LINES OF DISPLACEMENTS

Figure 3 presents the results of measurements of the displacements at the top of the shell due to the loader load, as it was discussed before in the example in the test of A4 [2]. On the horizontal axis there is given the location of the front axle of vehicle with reference to the top of the shell, i.e. when $x_p = 0$ this axle is over top of the shell, but the rear axle is over the coordinate – 3.4 m. On the account of the small span of the shell and the large track of axle of loader, on the graph there can be highlighted the shape of the influence line of displacement. This is the part of the displacement graph which was created on the left side of the shell top. The creation of the influence line of displacements from these measurements was possible because:

- the track of wheels is large with reference to the span of the shell

$$a_{12}/L = 3.40/6.04 = 0.563;$$

- pressure on the rear axle is smaller than that on the front one

$$P_1/P_2 = 221/69 = 3.203.$$

Because values in the range of $-5 < x_p < -3$ are small, one can consider that the chart of deflection due to this load in the range of $-5 < x_p < 0$ constitutes the influence line of deflection. Obviously, the accuracy of this chart is perturbed by the interaction between the rest of the wheels of the vehicle. If the load was the one-point force, the shape obtained would be the longitudinal section of the influence surface of deflection.

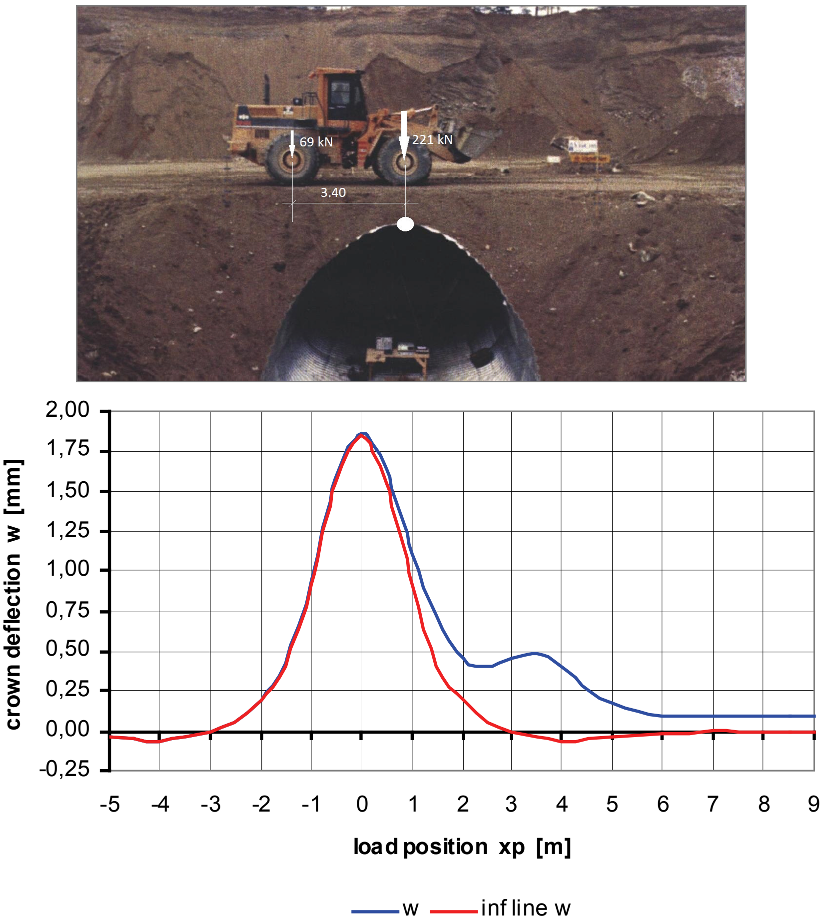

Fig. 3. Line of deflection obtained during the crossing of loader

Secondary extreme value of deflection shown in figure 3 is due to the load of the second axle of loader positioned at the top of the shell. Out of the ratio of deflections w_1/w_2 = 1.86/0.485 = 3.8 the result similar to the ratio of load P_1/P_2 can easily be seen. The chart shown in figure 3 does not approach the zero value of the deflection, i.e. w_r/w = 6.69% (compare table 2). In the case of a small value of thickness of the backfilling soil (H < 1.50 m), more favourable ratio of w_r/w can be obtained.

3. THE MODEL OF A STRUCTURE

Live loads on the road bridges are vehicles which are modelled as the group of point forces. Their reactions to the structure is through the layers of the road surface. The crucial problem in the design of bridges is to find such a position of the vehicle with reference to the element under consideration which gives the extreme (minimal or maximal) values of the internal forces. To achieve such an aim, the influence function referred to the road surface can be used. On the basis of the influence function obtained (line or surface) it can be possible to specify two types of information: the type of internal forces and the position of the considered point of the shell $S(x_a, y_a, z_a)$. The definition of the influence surface leads to the statement that the internal force in the shell S is calculated on the basis of the coordinate $\mu(x_a, x_p, y_a, y_p, z_a)$, created at the place where the point force is applied, over the roadway surface $P(x_p, y_p)$,with the following formula:

$$S(x_a, y_a, z_a) = \mu\,(x_a, x_p, y_a, y_p, z_a)\cdot P(x_p, y_p). \tag{2}$$

The model of the structure is the spatial one made by means of volume elements of soil and road surface as in figure 4a. To model the geometry of corrugated steel shell of the shape shown in figure 4b, the beam elements are applied as in figure 4c. The space frame model made of beam elements applied to the discretization of the shell enables us to create the influence function with the help of kinematic constraint, in the way analogous to that used for other bar structures [1], [3]. The application of the two-dimensional elements to the model of the corrugated steel shell as the orthotropic one can give equal results as in the case of the beam elements [1]. Thus, the influence function can be developed by means of the static method – based on the multiple solution of the set of equations of load (repeatedly solved) with one-point force over the roadway surface. The contact layer (interface) between the backfilling and the shell as in figure 4a is modelled as the beam elements with the linearly-elastic parameters. For making the calculations of the model, the general system of FEM is applied.

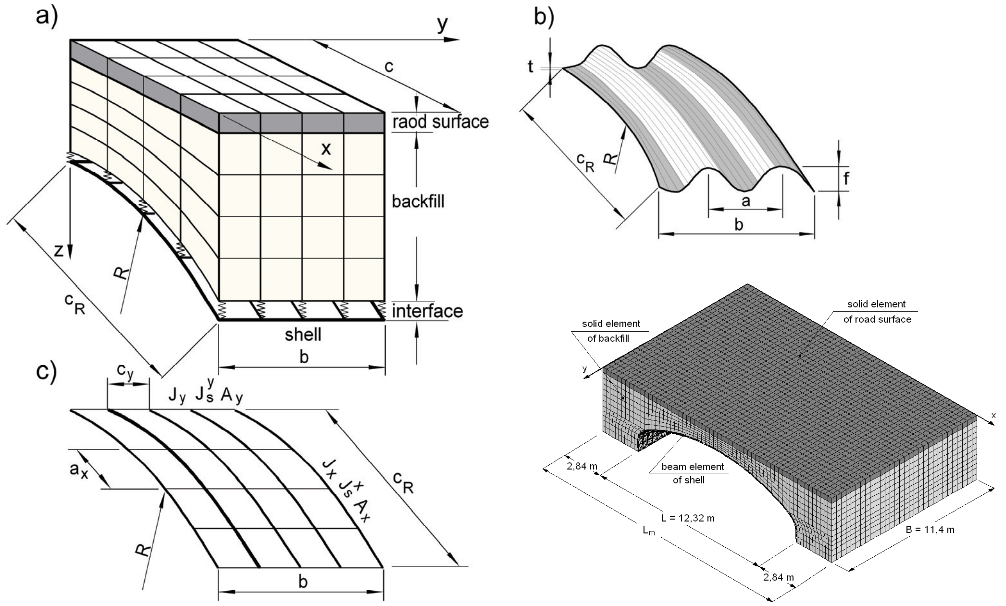

Fig. 4. The model of the structure and the shell created with the bar model

The shape of the influence surface is used to estimate the negative position of the group of forces over the roadway. The choice of the analyzed element in the considered type of structure as in figure 4c is optional.

4. THE KINEMATIC CONSTRAINT OF THE INFLUENCE FUNCTION

Among the few methods of creating the influence function of internal forces a very useful is that based on the kinematic constraint as the result of the basis deformation of the element analyzed [1], [3]. The place and the shape of that deformation are referred to the internal force S under consideration. This algorithm can be applied in any model of geometry, provided that the element under consideration in which the force S is estimated is the beam [3]. An essential rule of the kinematic method of creating the influence function is the estimation of vertical displacement of the road surface (corresponding to the direction of the external load) from the imposed deformation of kinematic constraint. Technically, it is realized in the model of structure geometry with the help of FEM as in figure 4, applying the substitute loading of the nodes of the elements as in figure 5, which causes the imposed deformation which referring to the internal force under consideration as in figure 6. The method of estimating the nodal forces to obtain internal force S is given in figure 5.

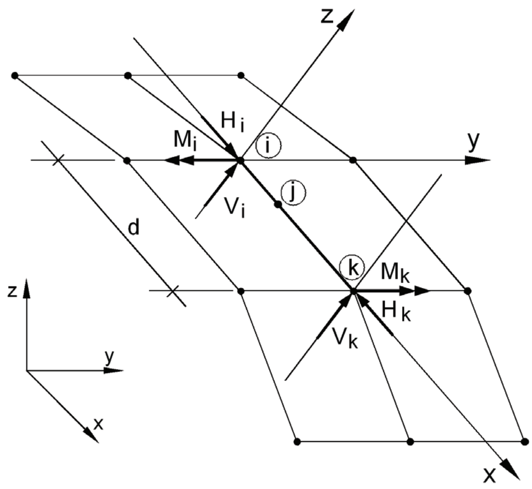

Fig. 5. Nodal loads of kinematic constraint of the element with nodes *ik*

To obtain the influence function of bending moment in the section of *j*, one has to cause a forced displacement of this node which corresponds to the internal force under consideration, i.e. $\varphi_j = 1$. The angle of deflection of the bar over the infinitely small section *dx* consists of the two parts which correspond to the sections *ij* and *jk*, as in figure 6a, with the total value

$$\varphi_{ji} + \varphi_{jk} = 1. \tag{3}$$

This deformation of the bar accompanies the displacement of the node *j* which is of unknown value w_j.

On both sides of the node *j* there is the conformity of the bending moments as in figure 5b

$$M_{ji} = M_{jk}, \tag{4}$$

whose values are calculated using the transformation formulas of displacement method on the basis of resultant deformation displacements shown in figure 6b and 6c

$$6\frac{EI}{c^2}w_j - 4\frac{EI}{c}\varphi_{ji} = 6\frac{EI}{b^2}w_j - 4\frac{EI}{b}\varphi_{jk}. \tag{5}$$

After the reduction of the equation of bending stiffness *EI* of the beam one obtains

$$\frac{3}{c^2}w_j - \frac{2}{c}\varphi_{ji} = \frac{3}{b^2}w_j - \frac{2}{b}\varphi_{jk}. \tag{6}$$

On both sides of the node *j* the shear forces are in conformity with one another, as in figure 6c

$$T_{ji} = T_{jk}. \tag{7}$$

They are calculated as in the case of bending moments, on the basis of the formula

$$12\frac{EI}{c^3}w_j - 6\frac{EI}{c^2}\varphi_{ji} = -12\frac{EI}{b^3}w_j + 6\frac{EI}{b^2}\varphi_{jk}\,. \tag{8}$$

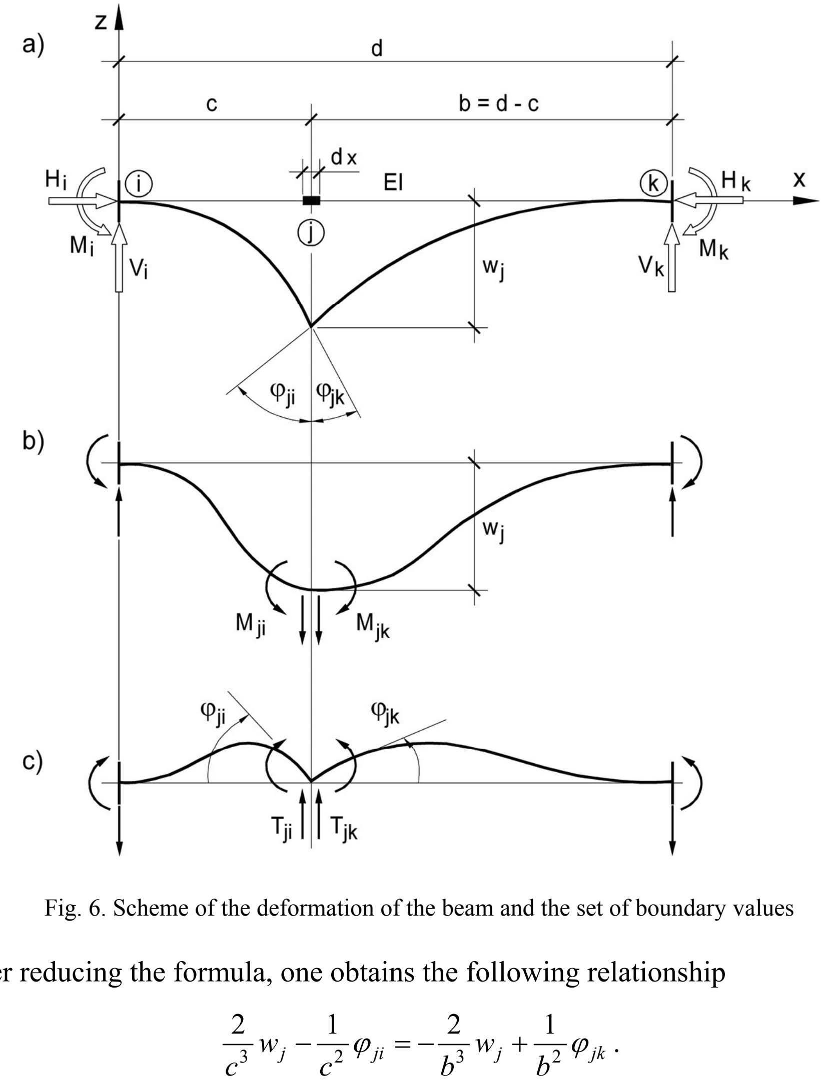

Fig. 6. Scheme of the deformation of the beam and the set of boundary values

After reducing the formula, one obtains the following relationship

$$\frac{2}{c^3}w_j - \frac{1}{c^2}\varphi_{ji} = -\frac{2}{b^3}w_j + \frac{1}{b^2}\varphi_{jk}\,. \tag{9}$$

Out of equations (3), (6) and (9) one obtains the displacements under considerations (figure 6a)

$$w_j = 2d\left(\frac{c}{d}\right)^2\left(\frac{b}{d}\right)^2,$$

$$\varphi_{ji} = \frac{c}{d}\left[1+3\frac{b}{d}\left(1-2\frac{c}{d}\right)\right], \tag{10}$$

$$\varphi_{jk} = \frac{b}{d}\left[1+3\frac{c}{d}\left(2\frac{c}{d}-1\right)\right].$$

They are used to calculate the boundary forces in the node *i* (figure 6a)

$$M_i = 6\frac{EI}{c^2}w_j - 2\frac{EI}{c}\varphi_{ji} = 2\left(2-3\frac{c}{d}\right)\frac{EI}{d},$$

$$V_i = 12\frac{EI}{c^3}w_j - 6\frac{EI}{c^2}\varphi_{ji} = 6\left(1-2\frac{c}{d}\right)\frac{EI}{d^2}, \tag{11}$$

and in the node *k*

$$M_k = 6\frac{EI}{b^2}w_j - 2\frac{EI}{b}\varphi_{ji} = 2\left(3\frac{c}{d}-1\right)\frac{EI}{d},$$

$$V_k = 12\frac{EI}{b^3}w_j - 6\frac{EI}{b^2}\varphi_{jk} = 6\left(2\frac{c}{d}-1\right)\frac{EI}{d^2}. \tag{12}$$

• To obtain the influence function of the axial force $S = N$ in the section *j* one has to cause a forced displacement of this node which corresponds to the internal force analysed, i.e. $\Delta d = 1$. The change of the length of the bar develops the internal forces with the same values but with the opposite directions

$$H_i = H_k = \frac{EA}{d}. \tag{13}$$

In the case of the influence function of axial force, the location of the section *j* is of no significance. In figure 6, there is presented the system of the nodal forces of kinematic constraint of the shell for arbitrary bar of the bar mesh of the shell model. The forces shown in figure 5 represent the circumferential strip of shell.

• On the basis of the axial force and bending moment in the considered section of the bar it is possible to calculate the normal stresses. Maximal values of normal stresses are obtained as follows [1]:

$$\sigma_{\max} = \frac{N}{A} + \frac{M}{2I}(f + t)\ . \tag{14}$$

Using (14) one can determine also the forces of kinematic constraint which can give the influence function of stresses $S = \sigma$. In the case where the mesh of elements is very dense, as in the structures analyzed, it is convenient to take the analyzed section j in the middle of the element. Then one has $c = b = d/2$ and $V_i = V_k = 0$. Also the formulas for boundary moments take more simplified form

$$M_i = M_k = \frac{EI}{d}\ . \tag{15}$$

When the influence function of maximum stresses is to be found in the circumferential strip of shell, as in figure 4c, in the element with the length of $d = a_x$ nodal forces take the values calculated from the formulas:

$$H_i = H_k = \frac{E}{d}\,, \qquad M_i = M_k = \frac{E}{2d}(f + t)\,, \tag{16}$$

where E is the modulus of elasticity of the shell and the remaining dimensions are given in figure 4b and 4c. When the influence surface of normal stresses is created, the nodal forces of kinematic constraint from the axial force N and the bending moment M occur simultaneously. As a result of calculations due to such a loading we obtain vertical deflections of roadway surface (corresponding to the direction of external loads) which form also the coordinates of the influence surface μ in equation (2). Other schemes of kinematic constraints and formulas which are of use in the calculations of nodal forces, as in figure 6, are given in [3].

5. INFLUENCE LINES AND SURFACES OF INTERNAL FORCES

Live loads of road bridges are usually vehicles, and their effect on the shell spreads through the layers of the road surface (and backfilling soil). The location of vehicle during normal service life can be arbitrary but usually limited to the curbs or crash barrier. To obtain the maximal values of internal forces in the analyzed elements of structure it is essential to place the live load on the surface of the bridge in an appropriate manner. To do so, one has to use the influence surface of internal forces in the shell, created on the surface of roadway on which the load is located.

A crucial point in the estimation of the load carrying capacity is to find such a section of the shell which is under the greater stress; usually it is located at the top or at the corner. In the case of steel bridges with the same strength in the whole circumferential strip and without additional cover plate, the choice of the place is not so difficult. For such elements (section) the influence surface of internal forces is created. The influence function calculated (influence line or surface) has to comprise the position of the considered point over the structure $A(x_a, y_a)$ and the type of internal force $S(x_a, y_a)$.

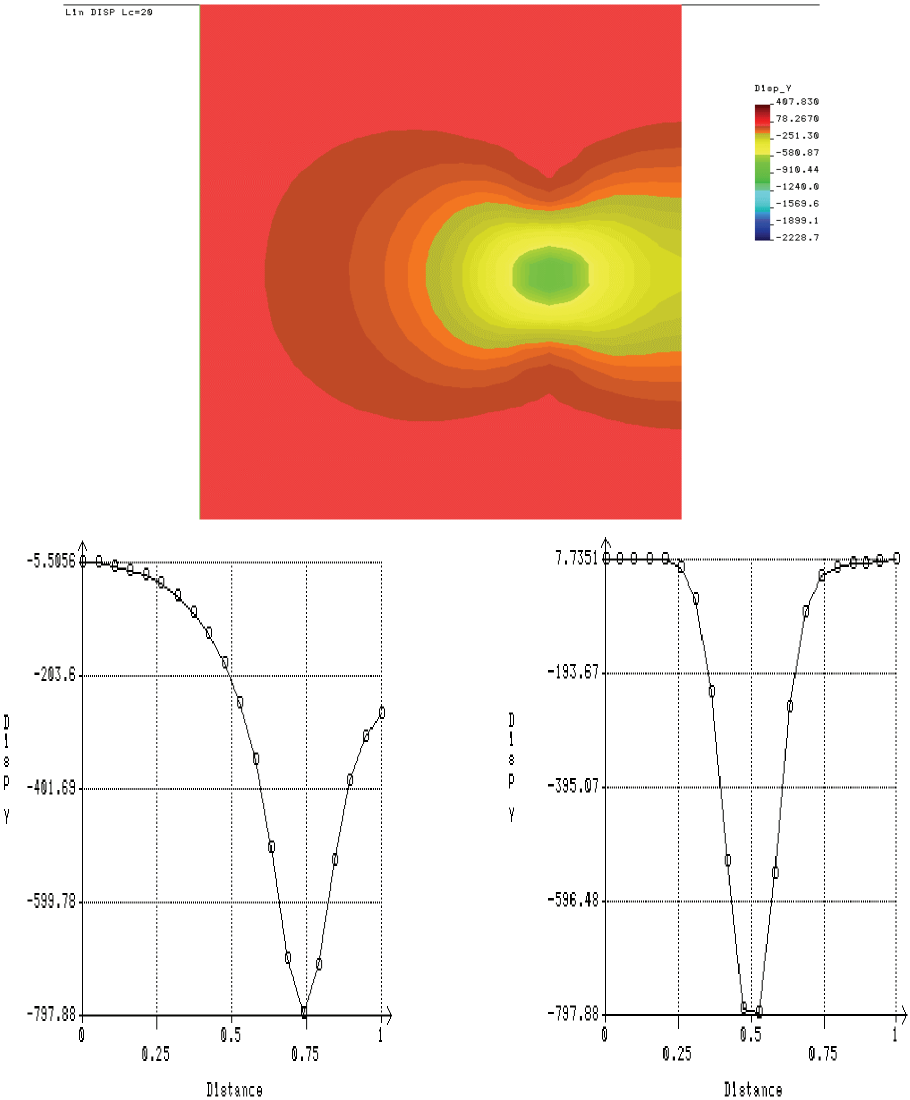

Fig. 7. Influence surface σ_g (0.0, 1.5)

In figure 7, there is presented the example of influence surface of normal stresses (maximum on the top wave) at the top, in the circumferential direction of shell. Geometrical parameters of the bridge structure shown in figure 4 are as follows: L = 5.655 m, B = 7.60 m, H = 0.55 m. The box-shaped shell is constructed with the plate of profile type MP 150×50×6, which is the bar structure with the mesh # 355 mm. A 3D model used was created as the result of numerical tests of numerous models of the shell surrounded by the backfilling and on the basis of site measurements of real structure [1]. The discretization of the geometry of the structure analyzed is based on:

- road surface as the isotropic continuum (E = 260 MPa and ν = 0.20);
- backfilling soil as orthotropic continuum (E_z = 220 MPa, $E_x = E_y$ = 60 MPa and ν = 0.20);
- contact layer (interface) modelled on the basis of one-dimensional elastic elements.

The forces of kinematic constrains for normal stress in the circumferential strip of the shell are calculated from equation (15):

$$H_i = H_k = \frac{205000}{0.355} = 577465 \ \text{MN/m}^3,$$

$$M_i = M_k = \frac{205000}{0.355} 0.056 = 16169 \ \text{MN/m}^2.$$

If the geometry of the bridge is described in meters and E in MN/m^2, then the forces N and M calculated from equation (14) are in MN/m^2, and the results of the calculations, i.e. deformations (deflections), which are also the coordinates of the influence surface μ_σ, are in m^{-2}. The numerical analysis was carried out with the help of the system COSMOS/M. Contour plot of normal stresses on the bottom and top edge $\sigma_g(x_0, y_0)$ (figure 7) has the vertical dimension (direction x) 6.75 m (the length of the structure with the part of ground) and the horizontal dimension equal to the width of road, i.e. 6.60 m. The location of the point of the shell O(x_0, y_0) under consideration is referred to the middle of the road surface and bridge and the coordinates are given in meters.

At the bottom part of figure 7, on the left side, there is given the cross section (along the y_0-axis) and on the right side – longitudinal section (along the x_0-axis). They allow mapping the shape of the influence surface. The reference point on the horizontal axis makes it possible to assess the position of the cross-section with reference to both the width of the bridge (the graph on the left side) and the span (the graph on the right side). In the case of the longitudinal section, one has to remember about extended model with respect to the span which results in the referring this profile to the middle of the span of bridge (coordinate 0.5). The analysis of the shape of influence surface is given in [1]–[3].

6. SUMMARY

If the model of the structure is made in 2D as the plain circumferential strip of the soil-steel structure, the influence line of the internal force under consideration being the displacement (vertical one) of the line of roadway surface is obtained as a result of the calculations based on kinematic constraint [1], [3]. In the case of the 3D model of the spatial object, there is obtained the influence surface of this internal force being the displacement (vertical one) of the road surface. The algorithm presented can be used in any FEM-based program. It does not require any changes in the discretization of the model during the choice of element under consideration. It is based on the simple formulas of the boundary forces.

The main area of applying the influence function is the calculation of internal forces in the shell, which are generated by live loads. It is used to determine the position of the group of forces of the vehicle load. In papers [4], [6], the influence functions have been used in parametric analysis for the estimation of the usefulness of the real shape and geometric parameters of the constructed structures. In this analysis, the thickness of the backfill soil above the shell and the stiffness of the corrugated plates of the shell are of a crucial importance.

Due to the application of the influence function in the above formulation, there has been presented the algorithm which enables application of 2D models in the form of circumferential strip of the structure [5]. Then the professional FEM software created especially for the computations of the soil-steel structures can be used.

REFERENCES

[1] MACHELSKI C., MICHALSKI J.B., JANUSZ L., *Deformation factors of buried corrugated structures*, Journal of the Transportation Research Board, Solid Mechanics, Transportation Research Board of Nationals Academies, Washington D.C., 2009, 70–75.

[2] PETTERSSON L., *Full Scale Tests and Structural Evaluation of Soil Steel Flexible Culverts with Low Height of Cover*, Doctoral Thesis in Civil and Architectural Engineering, KTH Stockholm, Sweden, 2007.

[3] MACHELSKI C., *Applications of kinematic method in the influence function of internal forces in the beam structures* (in Polish), *Zastosowanie metody kinematycznej do wyznaczania funkcji wpływu sił wewnętrznych w układach prętowych*, Inżynieria i Budownictwo, 1998, nr 7.

[4] MACHELSKI C., ANTONISZYN G., *Influence of live loads on the soil-steel bridges*, Studia Geotechnica et Mechanica, 2004, 3–4, 91–119.

[5] MACHELSKI C., ANTONISZYN G., *Load rate of the circumferential sector of soil-steel bridges structures*, Archives of Civil and Mechanical Engineering, 2005, Vol. V, No. 4. 85–102.

[6] ANTONISZYN G., *Soil-steel bridges. Influence of geometrical parameters on internal forces in steel shell* (in Polish), *Mosty gruntowo-powłokowe. Wpływ ukształtowania obiektu na siły wewnętrzne w stalowej powłoce*, MOSTY, 2008, nr 2, 26–35.