Studia Geotechnica et Mechanica, Vol. XXXII, No. 4, 2010

THERMAL INSTABILITY OF RIVLIN–ERICKSEN ELASTO-VISCOUS ROTATING FLUID PERMEATING WITH SUSPENDED PARTICLES UNDER VARIABLE GRAVITY FIELD IN POROUS MEDIUM

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Abstract: The problem of thermal instability of the Rivlin–Ericksen elasto-viscous fluid in a porous medium is considered in the presence of uniform rotation, suspended particles and variable gravity field. The rotation, gravity field, suspended particles and viscoelasticity introduce oscillatory modes. It is found that the principle of the exchange of stabilities is valid, provided that some condition is fulfilled. In a stationary convection, suspended particles are found to have destabilizing effect on the system, while rotation has stabilizing effect on the system under certain conditions. The effect of rotation, suspended particles, and medium permeability have also been shown graphically.

NOMENCLATURE

g – variable gravity,
K' – Stokes drag coefficient,
N – density number of suspended particles,
P – pressure,
T – temperature,
$q(u, v, w)$ – fluid velocity,
α – thermal coefficient of expansion,
β – temperature gradient,
δP – perturbation of pressure,
$\delta\rho$ – perturbation of density,
ε – medium porosity,
η' – particle radius,
κ – thermal diffusivity,
ζ – fluid density,
υ – kinematic viscosity,
υ' – kinematic viscoelasticity,
Θ – perturbation of temperature.

1. INTRODUCTION

The theoretical and experimental results of the onset of thermal instability (Bénard convection) in a fluid layer under conditions of varying hydrodynamic and hydromagnetic stability have been treated in detail by CHANDRASEKHAR [l]. LAPWOOD [2] has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by WOODING [3]. SHARMA and KUMAR [4] have studied the thermal instability of an Oldroydian viscoelastic fluid in porous medium and also have considered the effect of uniform rotation on the instability.

With a growing importance of non-Newtonian fluids in modern technology and industries, the investigations into such fluids are desirable. There are many elastoviscous fluids that can be characterised neither by Maxwell's constitutive relations nor by Oldroyd's constitutive relations. One such class of viscoelastic fluids is the Rivlin–Ericksen's fluid. RIVLIN and ERICKSEN [5] have proposed a theoretical model for such viscoelastic fluid. This and the other class of polymers are used for manufacturing parts of space-crafts, aeroplanes, tyres, belt conveyers, ropes, cushions, seats, foams, plastics, engineering equipments, etc. Recently, polymers are also used in agriculture, communication appliances and in biomedical applications.

When fluid permeated a porous material, the gross effect is represented by Darcy's law. As a result of this macroscopic law, the usual viscous terms in the equations of Rivlin–Ericksen's elasto-viscous fluid motion is replaced with

$$\left[-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial \vec{q}}{\partial t}\right)\right],$$

where μ and μ' are the viscosity and viscoelasticity in the thermal instability of the Rivlin–Ericksen fluid, k_1 is the medium permeability and $\vec{q}$ is the Darcian (filter) velocity of the fluid.

STOMMEL and FEDOROR [6] and LINDEN [7] have remarked that the length-scale characteristics of double diffusive convecting layers in the ocean may be sufficiently large to make the Earth rotation important to their formation. Moreover, the rotation of the Earth distorts the boundaries of a hexagonal convection cell in a fluid through a porous medium and the distortion plays an important role in the extraction of energy in the geothermal regions. The problem of thermal instability in fluid in porous mediums is of importance in geophysics, soil sciences, ground water hydrology and astrophysics. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat transfer mechanism in the development of young oceanic crust (LISTER [8]). Generally, it is accepted that comets consist of a dusty "snowball", a mixture of frozen gases, which in the process of its journey changes from solid to gas and vice versa. The physical properties of comets, meteorities and interplanetary dust strongly suggest the importance of porosity in the astrophysical material (McDONNEL [9]).

Thermal instability of a fluid layer under variable gravitational field heated from below or above is investigated analytically by PRADHAN and SAMAL [10]. Although the gravity field of the Earth is varying with the height from its surface, we usually neglect this variation in laboratory proposes and treat the field as constant. However, this may not be the case for large-scale flows in the ocean, the atmosphere or the mantle. It can become imperative to consider gravity as a quantity varying with the distance from the centre.

SHARMA and RANA [11] have studied the thermal instability of Walters' (Model B') elasto-viscous fluid in the presence of variable gravity field and rotation in porous medium. SHARMA and RANA [12] have also studied the instability of streaming the Rivlin–Ericksen fluid in porous medium in hydromagnetics. Recently, they [13] have studied the thermosolutal instability of the Rivlin–Ericksen rotating fluid in the presence of magnetic field and variable gravity field in a porous medium.

Keeping in mind the importance of ground water hydrology, soil sciences, geophysics and astrophysics, the thermal instability of the Rivlin–Ericksen elasto-viscous rotating fluid that permeates with suspended particles under variable gravity field in porous medium has been considered in this paper.

2. FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

Consider an infinite, horizontal, incompressible Rivlin–Ericksen viscoelastic fluid of the thickness d, bounded by the plane $z = 0$ and $z = d$ in an isotropic and homogeneous porous medium of the porosity ε and the medium permeability k_1, which is acted upon by a uniform rotation $\vec{\Omega}(0, 0, \Omega)$ and variable gravity $\vec{g}$ (0, 0, $-\vec{g}$), where $\vec{g} = \lambda g_0$, g_0 (>0) is the value of g at $z = 0$ and λ can be positive or negative as gravity increases or decreases from its value g_0. The layer is heated from below such that a uniform temperature gradient

$$\beta = \left(\left|\frac{dT}{dz}\right|\right)$$

is maintained.

Let P, ρ, T, α, υ, υ' and $\vec{q}(u, v, w)$ denote, respectively, pressure, density, temperature, thermal coefficient of expansion, kinematic viscosity, kinematic viscoelasticity, and velocity of the fluid. $K' = 6\pi\rho\upsilon\eta'$; η', being the particle radius, is the Stokes drag coefficient.

$$\dot{x} = (x, y, z), \quad E = \varepsilon + (1-\varepsilon)\frac{\rho_s C_s}{\rho_0 C_f}$$

which is constant, κ is thermal diffusivity, ρ_s, C_s, ρ_0, C_f denote the density and heat capacity of solid (porous) matrix and fluid, respectively; $\vec{q}_d(\bar{x}, t)$ and $N(\bar{x}, t)$ denote filter velocity and number density of the suspended particles, respectively, the suffix zero refers to the values at the reference level $z = 0$.

The equations of motion, continuity, heat conduction through porous medium and equation of the state of the Rivlin–Ericksen fluid are, respectively, as follows:

$$\frac{1}{\varepsilon}\left[\frac{\partial \vec{q}}{\partial t} + \frac{1}{\varepsilon}(\vec{q}.\nabla)\vec{q}\right] = -\left(\frac{1}{\rho_0}\right)\nabla P + \vec{g}\left(1 + \frac{\delta\rho}{\rho_0}\right)$$
$$-\frac{1}{k_1}\left(\upsilon + \upsilon'\frac{\partial}{\partial t}\right)\vec{q} + \frac{K'N}{\varepsilon\rho_0}(\vec{q}_d - \vec{q}) + \frac{2}{\varepsilon}(\vec{q}\times\Omega), \tag{1}$$

$$\nabla.\vec{q} = 0, \tag{2}$$

$$E\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T + \frac{mNC_{pt}}{\rho_0 C_t}\left[\varepsilon\frac{\partial}{\partial t} + \vec{q}_d.\nabla\right]T = \kappa\nabla^2 T\,, \tag{3}$$

$$\rho = \rho_0[1 - \alpha(T - T_0)]\,, \tag{4}$$

where the suffix zero refers to the values at the reference level $z = 0$.

Assuming a uniform size of particles, their spherical shape and low relative velocity between the fluid and particles, the presence of particles adds an extra force term in equation (1), proportional to the velocity difference between particles and the fluid. The force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid. The distances between particles are assumed quite large compared with their diameters, so an interparticle relation is ignored. The buoyancy force on the particles is neglected. If mN is the mass of the particle per unit volume, then, making the above assumptions, the equations of motion and continuity for the particles are as follows:

$$mN\left[\frac{\partial \vec{q}_d}{\partial t} + \frac{1}{\varepsilon}(\vec{q}_d.\nabla)\vec{q}_d\right] = K'N(\vec{q} - \vec{q}_d)\,, \tag{5}$$

$$\varepsilon\frac{\partial N}{\partial t} + \nabla.(N\vec{q}_d) = 0\,. \tag{6}$$

The initial state of the system is taken to be a quiescent layer (no settling) with the uniform particle distribution number. The initial state

$$\vec{q} = (0,0,0), \quad \vec{q}_d(0,0,0),$$
$$T = -\beta z + T_0, \quad \rho = \rho_0(1+\alpha\beta z), \tag{7}$$
$$N_0 = \text{constant}$$

is an exact solution to the governing equations. Let $\delta\rho$, δp, θ, $\vec{q}\,(u, v, w)$ and $\vec{q}_d(l, r, s)$ denote, respectively, the perturbation in the density ρ, the pressure P, the temperature T, the fluid velocity $\vec{q}$ = (0, 0, 0) and the particle velocity $\vec{q}_d(0,0,0)$. The change in the density $\delta\rho$ caused by the perturbation θ of temperature is given by

$$\delta\rho = -\alpha\rho_0\theta\,. \tag{8}$$

Then the linearized perturbation equations relevant to the problem are

$$\frac{1}{\varepsilon}\frac{\partial\vec{q}}{\partial t} = -\frac{1}{\rho_0}\nabla(\delta\rho) - \vec{g}(\alpha\theta) - \frac{1}{k_1}\left(\upsilon + \upsilon'\frac{\partial}{\partial t}\right)\vec{q} + \frac{K'N}{\varepsilon\rho_0}(\vec{q}_d - \vec{q}) + \frac{2}{\varepsilon}(\vec{q}\times\Omega) \tag{9}$$

$$\nabla.q = 0, \tag{10}$$

$$\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\vec{q}_d = \vec{q}\,, \tag{11}$$

$$(E + b\varepsilon)\frac{\partial\theta}{\partial t} = \beta(w + bs) + \kappa\nabla^2\theta\,, \tag{12}$$

where $b = (mNC_{pt})/(\rho C_f)$ and w, s are the vertical fluid and the particles' velocity.

In the Cartesian form, equations (9)–(12) with the help of equation (8) can be expressed as

$$\frac{1}{\varepsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial x}(\delta p)$$
$$-\frac{1}{k_1}\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\vec{u} - \frac{mN}{\varepsilon\rho_0}\frac{\partial u}{\partial t} + \frac{2}{\varepsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega v, \tag{13}$$

$$\frac{1}{\varepsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial v}{\partial t} = -\frac{1}{\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial y}(\delta\rho)$$
$$-\frac{1}{k_1}\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)v - \frac{mN}{\varepsilon\rho_0}\frac{\partial v}{\partial t} - \frac{2}{\varepsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega u, \tag{14}$$

$$\frac{1}{\varepsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial w}{\partial t}=-\frac{1}{\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial z}(\delta p)$$
$$-\frac{1}{k_1}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)w+g\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\alpha\theta-\frac{mN}{\varepsilon\rho_0}\frac{\partial w}{\partial t}, \tag{15}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{16}$$

$$(E+b\varepsilon)\frac{\partial\theta}{\partial t}=\beta(w+bs)+\kappa\nabla^2\theta. \tag{17}$$

Operating equations (13) and (14) by $\partial/\partial x$ and $\partial/\partial y$, respectively, adding and using equation (16), we arrive at

$$\frac{1}{\varepsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right)=\frac{1}{\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\nabla^2-\frac{\partial^2}{\partial z^2}\right)\delta p$$
$$-\frac{1}{k_1}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)\left(\frac{\partial w}{\partial z}\right)-\frac{mN}{\varepsilon\rho_0}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right)-\frac{2}{\varepsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega\zeta, \tag{18}$$

where

$$\zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$$

is the z-component of vorticity.

Operating equations (15) and (18) by

$$\left(\nabla^2-\frac{\partial^2}{\partial z^2}\right) \text{ and } \frac{\partial}{\partial z},$$

respectively, and adding to eliminate δp between equations (15) and (18), we obtain

$$\frac{1}{\varepsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial t}(\nabla^2 w)=-\frac{1}{k_1}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)\nabla^2 w$$
$$+\vec{g}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\alpha\theta-\frac{mN}{\varepsilon\rho_0}\frac{\partial}{\partial t}(\nabla^2 w)-\frac{2}{\varepsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega\frac{\partial\zeta}{\partial z}, \tag{19}$$

where

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}.$$

Operating equations (13) and (14) by

$$-\frac{\partial}{\partial y} \quad \text{and} \quad \frac{\partial}{\partial x},$$

respectively, and adding, we have

$$\frac{1}{\varepsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial t}(\zeta)=-\frac{1}{k_1}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)\zeta$$
$$-\frac{mN}{\varepsilon\rho_0}\frac{\partial}{\partial t}(\zeta)+\frac{2}{\varepsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega\frac{\partial w}{\partial z}. \tag{20}$$

3. THE DISPERSION RELATION

Analyzing the disturbances of normal mode we assume that the perturbation quantities are of the form

$$[w, s, \theta, \zeta]=[W(z), S(z), \Theta(z), Z(z)]\exp(ik_x x+ik_y y+nt), \tag{21}$$

where k_x, k_y are the wave numbers along the x and y directions, respectively, $k = (k_x^2+k_y^2)^{1/2}$ is the resultant wave numbers and n stands for the growth rate which is, in general, a complex constant.

Using expression (21) in (l9), (20) and (17) we obtain

$$\frac{n}{\varepsilon}\left[\frac{d^2}{dz^2}-k^2\right]W=-gk^2\alpha\Theta-\frac{1}{k'}(\upsilon+\upsilon' n)\left(\frac{d^2}{dz^2}-k^2\right)W$$
$$-\frac{mNn}{\varepsilon\rho_0\left(\dfrac{m}{K'}n+1\right)}\left(\frac{d^2}{dz^2}-k^2\right)W-\frac{2\Omega}{\varepsilon}\frac{dZ}{dz}, \tag{22}$$

$$\frac{n}{\varepsilon}Z=-\frac{1}{k'}(\upsilon+\upsilon' n)Z-\frac{mNn}{\varepsilon\rho_0\left(\dfrac{m}{K'}n+1\right)}Z+\frac{2}{\varepsilon}\Omega\frac{dW}{dz}, \tag{23}$$

$$(E+b\varepsilon)n\Theta=\beta(W+bs)+\kappa\left(\frac{d^2}{dz^2}-k^2\right)\Theta. \tag{24}$$

Equations (22)–(24) in nondimensional form become

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{(\tau_1\sigma+1)}\right)+\left(\frac{1+F\sigma}{P_t}\right)\right](D^2-a^2)W+g\frac{a^2d^2\alpha\Theta}{\upsilon}+\frac{2\Omega d^3}{\varepsilon\upsilon}DZ=0, \qquad (25)$$

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{(\tau_1\sigma+1)}\right)+\frac{1+F\sigma}{P_l}\right]Z=\frac{2\Omega d}{\varepsilon\upsilon}DW, \qquad (26)$$

$$[(D^2-a^2)-E_1P_1\sigma]\,\Theta=\frac{-\beta d^2}{\kappa}\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W, \qquad (27)$$

where we have put

$$a=kd,\quad \sigma=\frac{nd^2}{\upsilon},\quad \tau=\frac{m}{k^1},\quad \tau_1=\frac{\tau_1\upsilon}{d^2},\quad M=\frac{mN}{\rho_0}.$$

$E_1 = E + b\varepsilon$, $B = b + 1$, $F = \upsilon'/d^2$, $D^* = d(d/dz) = dD$, and the superscript * is suppressed, $P_t = \upsilon/\kappa^{-1}$ is the thermal Prandtl number, $P_l = k_1/d^2$ is the dimensionless medium permeability.

Eliminating Θ and Z between equations (25)–(27), we obtain:

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{(\tau_1\sigma+1)}\right)+\left(\frac{1+F\sigma}{P_t}\right)\right](D^2-a^2)(D^2-a^2-E_1p_1\sigma)W-Ra^2\lambda\left[\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right]W$$

$$+\left[\frac{\frac{T_A}{\varepsilon^2}(D^2-a^2-E_1p_1\sigma)}{\frac{\sigma}{\varepsilon}\left(1+\frac{M}{(\tau_1\sigma+1)}\right)+\left(\frac{1+F\sigma}{P_l}\right)}\right]D^2W=0\,, \qquad (28)$$

where

$$R=g_0\frac{\alpha\beta d^4}{\upsilon\kappa}$$

is the thermal Rayleigh number, and

$$T_A=\left(\frac{2\Omega d^2}{\upsilon}\right)^2$$

is the Taylor number.

Here we assume that the temperature at the boundaries is kept fixed, the fluid layer is comprised between two boundaries, and the adjoing medium is electrically nonconducting. The boundary conditions appropriate to the problem are

$$W = D^2W = DZ = \Theta = 0 \quad \text{at} \quad z = 0 \text{ and } 1 . \tag{29}$$

Using the boundary condition (29), we can show that all the even-order derivative of W must vanish on the boundaries and hence the proper solution of equaiton (28) characterizing the lowest mode is

$$W = W_0 \sin \pi z ; \tag{30}$$

W_0 is a constant.

Inserting equation (30) into (28) we obtain the dispersion relation:

$$R_1 x\lambda = \left[\frac{i\sigma_1}{\varepsilon}\left(1+\frac{M}{\pi^2\tau_1 i\sigma_1+1}\right)+\frac{1}{P}(1+\pi^2 Fi\sigma_1)\right]\times(1+x)(1+x+E_1p_1i\sigma_1)$$

$$\times\left(\frac{1+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right)+\frac{\frac{T_{A_1}}{\varepsilon^2}(1+x+Ep_1i\sigma_1)}{\frac{i\sigma_1}{\varepsilon}\left(1+\frac{M}{\pi^2\tau_1 i\sigma_1+1}\right)+\frac{1}{P}(1+\pi^2 Fi\sigma_1)}\times\left(\frac{1+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right), \tag{31}$$

where:

$$R_1 = \frac{R}{\pi^4},\quad T_{A_1} = \frac{T_A}{\pi^4},\quad x = \frac{a^2}{\pi^2},\quad i\sigma_1 = \frac{\sigma}{\pi^2},\quad P = \pi^2 P_i.$$

Equation (31) is the required dispersion relation accounting for the effect of suspended particles, medium permeability, variable gravity field, and rotation on the thermal instability of Rivlin–Ericksen fluid in porous medium.

4. STABILITY OF THE SYSTEM AND OSCILLATORY MODE

Here we examine the possibility of oscillatory mode, if any, in the Rivlin–Ericksen elasto-viscous fluid due to the presence of suspended particles, rotation, viscoelasticity and variable gravity field. Multiplying equation (25) by W^*, the complex conjugate of W, integrating over the range of z and making use of equations (26), (27) with the help of boundary conditions (29), we obtain

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{(\tau_1\sigma+1)}\right)+\frac{1+F\sigma}{P_t}\right]I_1-\frac{\lambda g_0\alpha a^2\kappa}{\upsilon\beta}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)\times(I_2+E_1P_1\sigma^* I_3)$$

$$-d^2\left[\frac{\sigma^*}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma^*}\right)+\frac{1+F\sigma^*}{P_t}\right]\int I_4 = 0, \tag{32}$$

where:

$$I_1 = \int_0^1 (|DW|^2 + a^2 |W|^2) dz,$$

$$I_2 = \int_0^1 (|D\Theta|^2 + a^2 |\Theta^2|) dz,$$

$$I_3 = \int_0^1 |\Theta|^2 dz,$$

$$I_4 = \int_0^1 |Z|^2 dz.$$

The integral parts I_1–I_4 are all positive definite. Inserting $\sigma = i\sigma_i$ into equation (32), where σ_i is real, and equating the imaginary parts we obtain

$$\sigma_i \left\{ \left[\frac{1}{\varepsilon}\left(1 + \frac{M}{1+\tau_1^2\sigma_i^2}\right) + \frac{F}{P_t} \right](I_1 + d^2 I_4) + \frac{\lambda g_0 \alpha a^2 \kappa}{\upsilon\beta}\left[\frac{\tau_1(B-1)}{B^2+\tau_1^2\sigma_i^2} I_2 + \frac{B+\tau_1^2\sigma_i^2}{B^2+\tau_1^2\sigma_i^2} p_1 E_1 I_3 \right]. \tag{33}$$

Equation (33) implies that $\sigma_i = 0$ or $\sigma_i \neq 0$ which means that mode may be nonoscillatory or oscillatory. The oscillatory modes were introduced due to the presence of rotation, gravity field and viscoelasticity.

5. EFFECT OF ROTATION

Equation (33) must be satisfied at the marginal state since $\sigma_i = 0$. Now for the principle of exchange of stability to be valid at marginal state, we must have $\sigma_i = 0$, which implies that the terms in the bracket in equation (33) must be positive definite. To prove this, equation (26) and $\sigma_i = 0$ yield

$$\frac{1}{P_t} Z = \left(\frac{2\Omega d}{\varepsilon\upsilon} \right) DW. \tag{34}$$

Multiplying both sides of equation (34) by Z^*, the complex conjugate of Z, integrating the resulting equation over the verticle range of Z and separating the real parts of both the sides of equation so obtained, we arrive at:

$$\frac{1}{P_i}\int_0^1 ZZ^* dz = Re\left(\frac{2\Omega d}{\varepsilon\upsilon}\right)\int_0^1 Z^* DW dz = \left(\frac{2\Omega d}{\varepsilon\upsilon}\right) Re\int_0^1 Z^* DW dz, \tag{35}$$

$$Re\int_0^1 Z^* DW dz \le \left|\int_0^1 Z^* DW dz\right| \le \int_0^1 |Z^*||DW| dz$$
$$\le \int_0^1 |Z||DW| dz \le \sqrt{\int_0^1 |Z|^2 dz}\sqrt{\int_0^1 |DW|^2 dz}. \tag{36}$$

Equations (35) and (36) give

$$\frac{1}{P_t}\int_0^1 |Z|^2 dz \le \frac{2\Omega d}{\varepsilon\upsilon}\sqrt{\int_0^1 |Z|^2 dz}\sqrt{\int_0^1 |DW|^2 dz},$$

which implies

$$\frac{1}{P_t}\int_0^1 |Z|^2 dz \le \left(\frac{2\Omega d}{\varepsilon\upsilon}\right)^2 P_t\int_0^1 |DW|^2 dz, \tag{37}$$

$$\int_0^1 |DW|^2 dz + a^2 |W|^2\, dz \ge \int_0^1 |DW|^2 dz. \tag{38}$$

Making use of inequalities (37) and (38) in equation (33) we have

$$\left[\frac{1}{\varepsilon}(1+M)+\frac{F}{P_t}\right](I_1+d^2 I_4)+\frac{\lambda g_0\alpha a^2\kappa}{\upsilon\beta}\left[\frac{\tau_1(B-1)}{B^2}\right]I_2+\frac{1}{B}p_1 E_1 I_3$$
$$\ge\left[\frac{1}{\varepsilon}(1+M)+\frac{F}{P_t}\right]\left[\int_0^1 |DW|^2 dz + d^2\left(\frac{2\Omega d}{\varepsilon\upsilon}\right)^2 P_t^2\int_0^1 |DW|^2 dz\right]$$
$$-\frac{\lambda g_0\alpha a^2\kappa}{\upsilon\beta}\left(\frac{\tau_1(B-1)}{B^2}\right)I_2+\frac{1}{B}p_1 E_1 I_3, \tag{39}$$

where

$$T_4 = \left(\frac{2\Omega d^2}{\varepsilon\upsilon}\right)^2$$

is the modified Taylor number.

But

$$\left[\left(\frac{1}{\varepsilon}(1+M)+\frac{F}{P_t}(1+T_A P_t^2)\right)\int_0^1 |DW|^2 dz+\frac{\lambda g_0 \alpha a^2 \kappa}{\upsilon\beta}\left[\frac{\tau_1(B-1)}{B^2}\right]I_2+\frac{1}{B}p_1 E_1 I_3>0, \quad (40)$$

if $B > 1$.

Thus equations (39) and (40) imply that for $B > 1$, when gravity increases from its value g_0, the term in the bracket in equation (33) its positive definite, which implies that $\sigma_i = 0$ and hence the necessary condition for the validity of the principle of the exchange of stabilities in the thermal instability of the Rivlin–Ericksen viscoelastic fluid in porous medium in the presence of rotation and a variable gravitational field, when gravity is increasing upward from its value g_0 (if $\lambda > 0$), is $B > 1$.

6. THE STATIONARY CONVECTION

For stationary convection putting $\sigma = 0$ in equation (41) reduces it to

$$R_1 x\lambda=\frac{1}{P}(1+x)(1+x)\times\frac{1}{B}+\frac{\frac{T_{A_1}}{\varepsilon^2}(1+x)}{\frac{1}{P}}\times\frac{1}{B}=(1+x)\left[\frac{(1+x)}{P}+\frac{T_{A_1}}{\varepsilon^2}P\right]\times\frac{1}{B}, \quad (41)$$

$$R_1=\frac{1+x}{x\lambda}+\left[\frac{1+x}{P}+\frac{T_{A_1}}{\varepsilon^2}P\right]\times\frac{1}{B}, \quad (42)$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters T_{A_1}, B and P. The Rivlin–Eriksen visoelastic fluid behaves like an ordinary Newtonian fluid since a viscoelastic parameter F vanishes with σ.

To study the effects of suspended particles, rotation and medium permeability, we examine analytically the behaviour of

$$\frac{dR_1}{dB}, \quad \frac{dR_1}{dT_{A_1}}, \quad \frac{dR_1}{dP}.$$

Equation (42) yields

$$\frac{dR_1}{dB}=-\left[\frac{1+x}{x\lambda}\left(\frac{1+x}{P}+\frac{T_{A_1}}{\varepsilon^2}P\right)\right]\times\frac{1}{B}, \quad (43)$$

which is negative, implying thereby that the effect of suspended particles is to destabilize the system when gravity increases upwards (i.e., $\lambda < 0$).

From equation (42) we get

$$\frac{dR_1}{dT_{A_1}} = \frac{P(1+x)}{\varepsilon^2 x\lambda B}, \tag{44}$$

which shows that rotation has a stabilizing effect on the system in porous medium when gravity increases upwards from its value g_0 (i.e., $\lambda > 0$).

Also from equation (42) we have

$$\begin{gathered}\frac{dR_1}{dP} = \frac{1+x}{x\lambda}\left[-\frac{1+x}{P^2} + \frac{T_{A_1}}{\varepsilon^2}\right] \times \frac{1}{B}, \\ \frac{dR_1}{dP} = -\frac{1+x}{Bx\lambda}\left[\frac{1+x}{P^2} - \frac{T_{A_1}}{\varepsilon^2}\right].\end{gathered} \tag{45}$$

From equations (45) we observe that medium permeability has destabilizing effect when

$$\frac{1+x}{P^2} > \frac{T_{A_1}}{\varepsilon^2}$$

and medium permeability has a stabilizing effect when

$$\frac{1+x}{P^2} < \frac{T_{A_1}}{\varepsilon^2},$$

when gravity increases upwards.

In the absence of rotation and for constant gravity field, dR_1/dP is always negative, implying thereby the destabilizing effect of medium permeability.

The dispersion relation (42) is analyzed numerically. Graphs have been plotted by giving some numerical values to the parameters, to depict the stability characteristics.

In figure 1, R_1 is plotted against B for $\lambda = 2$, $P = 0.2$, $T_{A_1} = 5$ and $\varepsilon = 0.5$ for fixed wave numbers $x = 0.5$ and $x = 0.8$. For wave numbers $x = 0.5$ and $x = 0.8$, the suspended particles have a destabilizing effect. In figure 2, R_1 is plotted against T_{A_1} for $\lambda = 2$, $P = 0.2$, $B = 3$, $\varepsilon = 0.5$ for fixed wave numbers $x = 0.5$ and $x = 0.8$. This shows that rotation has a stabilizing effect for fixed wave numbers $x = 0.5$ and $x = 0.8$.

In figure 3, R_1 is plotted against P for $\lambda = 2$, $\varepsilon = 0.5$, $T_{A_1} = 5$ and $B = 3$ for fixed wave numbers $x = 0.5$ and $x = 0.8$. This shows that medium permeability has a destabilizing effect for $P = 0.1$ to 0.3 and has a stabilizing effect for $P = 0.3$ to 1.0 for fixed wave numbers $x = 0.5$ and $x = 0.8$.

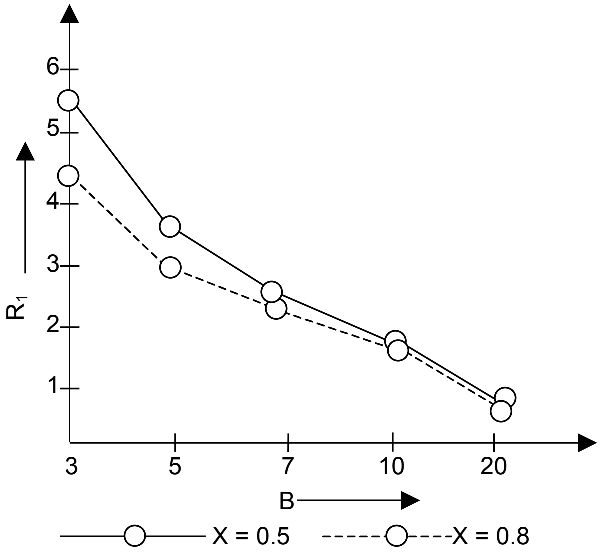

Fig. 1. Variation of the Rayleigh number R_1 with suspended particles B for $\lambda = 2$, $P = 0.2$, $T_{A_1} = 5$ and $\varepsilon = 0.5$ for fixed wave numbers $x = 0.5$ and $x = 0.8$

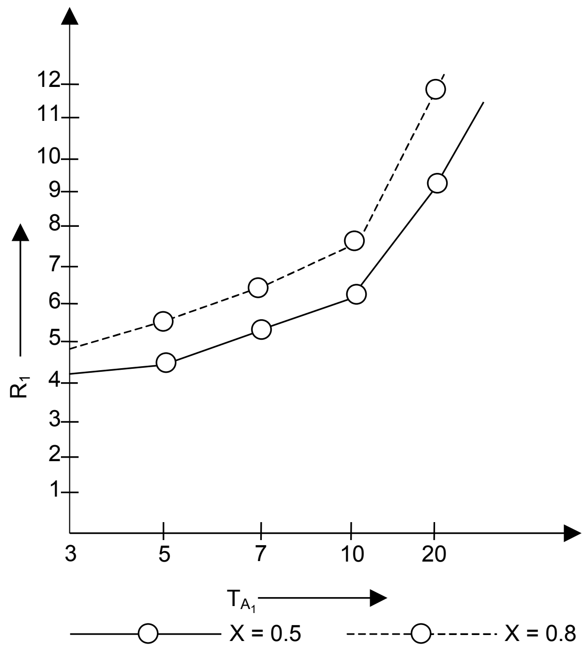

Fig. 2. Variation of the Rayleigh number R_1 with rotation T_{A_1} for $\lambda = 2$, $P = 0.2$, $B = 3$, $\varepsilon = 0.5$ for fixed wave numbers $x = 0.5$ and $x = 0.8$

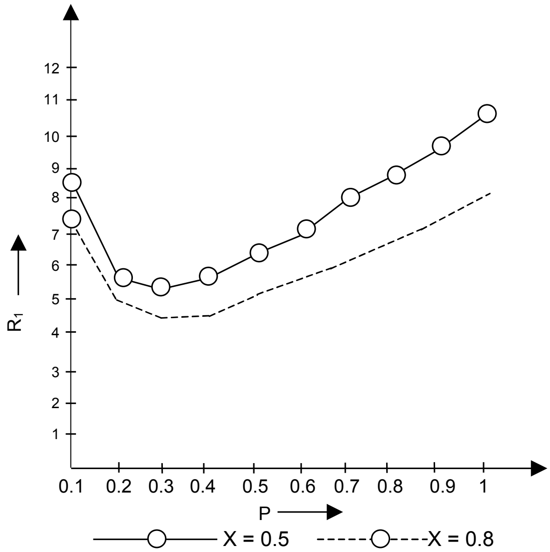

Fig. 3. Variation of the Rayleigh number R_1 with medium permeability P for $\lambda = 2$, $\varepsilon = 0.5$, $T_{A_1} = 5$ and $B = 3$ for fixed wave numbers $x = 0.5$ and $x = 0.8$

7. CONCLUSION

For the stationary convection, the rotation has a stabilizing effect for $\lambda > 0$ and a destabilizing effect for $\lambda < 0$ which is in contrast to the Newtonian fluids. The suspended particles are found to have a destabilizing effect on the system as gravity increases upward from its value g_0 (i.e., for $\lambda > 0$) and a stabilizing effect as gravity decreases upwards from its value g_0 (i.e., for $\lambda < 0$), whereas the medium permeabitity has a destabilizing/stabilizing effect on the system for

$$\frac{1+x}{P^2} > \frac{T_{A_1}}{\varepsilon^2}, \quad \frac{1+x}{P^2} < \frac{T_{A_1}}{\varepsilon^2} \quad \text{for} \quad \lambda > 0 .$$

Rotation, gravity field, suspended particles and viscoelasticity introduce oscillatory modes. The effects of the rotation, suspended particles and medium permeability on thermal instability have been shown graphically.

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