

STABILITY OF STRATIFIED OLDROYDIAN FLUID IN HYDROMAGNETICS IN PRESENCE OF SUSPENDED PARTICLE IN POROUS MEDIUM

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Abstract: In the present note, the stability of stratified Oldroyd viscoelastic fluid in the presence of suspended particles and variable magnetic field in porous medium has been studied. The case of exponentially varying density, viscosity, magnetic field and suspended particles is considered. The system is found to be unstable for the disturbances of all wave numbers for potentially unstable stratification. The effect of variable horizontal, magnetic field has been discussed in the note. The growth rate decreases or increases with the increase in particle density and kinematic viscosity; similarly, the growth rate increases or decreases with an increase in medium permeability and strain retardation time.

1. INTRODUCTION

The stability of an incompressible heavy fluid of variable density was investigated by Lord Rayleigh and is termed as the Rayleigh–Taylor instability. Several authors have studied the Rayleigh–Taylor instability of Newtonian fluid with varying assumptions of hydrodynamics and hydromagnetics and a detailed account of these results has already been given in the celebrated monograph of CHANDRASHEKHAR [1]. The medium in these problems has been considered to be nonporous. The flow through porous medium has been of a considerable interest in recent years, particularly among petroleum engineers and geophysical fluid dynamicists [2]. In geophysical situations, more often the fluid is not pure but may be instead permeated in the suspended (or dust) particles. Thus, the problem of the Rayleigh–Taylor instability in fluids in a porous medium is of a great importance in geophysics, soil sciences, ground water hydrology and astrophysics. Generally, it is accepted that comets consist of a dusty ‘snowball’ of a mixture of frozen gases. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of the porosity in astrophysical context (McDONNELL [3]). In stellar interiors and atmospheres, variable and non uniform magnetic field acts and affects the nature of instability. With the growing importance of non-Newtonian fluid in industries, chemical technology and

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geographical fluid dynamics much attention is being paid to the investigation of such fluids. SUNIL and CHAND [4] have studied the Rayleigh–Taylor instability of a plasma in the presence of a variable magnetic field and suspended particles in porous media. Also SHARMA and PRADEEP KUMAR [5] have studied the Rayleigh–Taylor instability of Oldroydian viscoelastic fluids in porous medium in the presence of variable magnetic field.

The stability of shear flow of a heterogeneous or stratified fluid has been considered by many research workers. Recently, the stability of stratified Rivlin–Ericksen viscoelastic fluid in the presence of suspended particles and variable magnetic field in porous medium has been studied by SUNIL, SHARMA and RAJENDER SINGH CHANDEL [6], while the stability of stratified Walters (Model B') viscoelastic fluid in stratified porous medium is given by SUNIL, DIVYA, SHARMA and VEENA SHARMA [7].

To the best of our knowledge, the stability of stratified Oldroydian viscoelastic fluid in the presence of suspended particles and variable magnetic field in porous medium has not been investigated so far. So keeping in mind the importance of non-Newtonian fluids in modern technology and their various applications mentioned above the present paper is devoted to the consideration of the stability of stratified Oldroydian viscoelastic fluid in the presence of suspended particles and variable magnetic field in porous medium.

2. FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

Let T_{ij} , τ_{ij} , e_{ij} , δ_{ij} , P , q_i , x_i , μ , λ and $\lambda_0 (< \lambda)$ denote, respectively, the total stress tensor, shear stress tensor, rate of strain tensor, Kronecker delta, scalar pressure, velocity, position vector, viscosity, stress relaxation time and strain retardation time. Then the Oldroydian viscoelastic fluid is described by the constitutive relations

$$\begin{aligned} T_{ij} &= -P\delta_{ij} + \tau_{ij}, \\ \left(1 + \lambda \frac{d}{dt}\right)\tau_{ij} &= 2\mu \left(1 + \lambda_0 \frac{d}{dt}\right)e_{ij}, \\ e_{ij} &= \frac{1}{2} \left(\frac{\partial q_i}{\partial x_j} + \frac{\partial q_j}{\partial x_i} \right), \end{aligned} \quad (1)$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \bar{q} \cdot \nabla$$

is the ‘convective derivative’. Relations of the type (1) were proposed and studied by Oldroyd. He also showed that many rheological equations of state, of general validity,

reduce to (1) when linearized $\lambda_0 = 0$ gives the fluid to be Maxwellian, whereas $\lambda = \lambda_0 = 0$ yields the Newtonian viscous fluid. When a fluid flows through a homogeneous and isotropic porous medium, the gross effect is represented by Dracy's law. As a result, the usual viscous term is replaced by the resistance term

$$-\left(\frac{\mu}{k_1}\right)\bar{v},$$

where k_1 and \bar{v} denote, respectively, the medium permeability and the filter velocity.

Here we consider a static state in which an incompressible, electrically conducting Oldroydian fluid layer of variable density is arranged in horizontal strata in the presence of suspended particles and a variable horizontal magnetic field. The pressure P and density ρ are functions of vertical z -coordinate only. The character of the equilibrium of this initial state is determined by supposing that the system is slightly disturbed and then by following its further evolution. The fluid is under the action of the gravity $\bar{g}(0, 0, -g)$ and the magnetic field $\bar{H} = (H_0(z), 0, 0)$. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of the porosity ε and medium permeability k_1 .

Let P , ρ , μ and $\bar{v}(u, v, w)$ denote, respectively, the pressure, the density, the viscosity and the velocity of pure fluid, $\bar{u}(\bar{x}, t)$ and $N(\bar{x}, t)$ denote the filter velocity and the number density of the suspended particles. $K = 6\pi\mu\eta$, where η is the particle radius, is the Stokes drag coefficient. $\bar{u} = (l, r, s)$ and $\bar{x} = (x, y, z)$. Let g , ε and k_1 stand for acceleration due to gravity, medium porosity, medium permeability. Let μ_e denote magnetic permeability. Then the equations of motion and continuity for Oldroydian viscoelastic fluid with suspended particle and horizontal variable magnetic field in porous medium are

$$\begin{aligned} & \frac{\rho}{\varepsilon} \left(1 + \lambda \frac{\partial}{\partial t}\right) \left[\frac{\partial \bar{v}}{\partial t} + (\bar{v} \cdot \nabla) \bar{v} \right] \\ & = \left(1 + \lambda \frac{\partial}{\partial t}\right) \left[-\nabla P + \bar{g}\rho + \frac{\mu e}{\pi} \{(\nabla \times \bar{H}) \times \bar{H}\} + \frac{KN}{\varepsilon} (\bar{u} - \bar{v}) \right] - \frac{\mu}{k_1} \left(1 + \lambda_0 \frac{\partial}{\partial t}\right) \bar{v}, \quad (2) \end{aligned}$$

$$\nabla \cdot \bar{v} = 0, \quad (3)$$

$$\nabla \cdot \bar{H} = 0, \quad (4)$$

$$\varepsilon \frac{\partial \bar{H}}{\partial t} = (\bar{H} \cdot \nabla) \bar{v} - (\bar{v} \cdot \nabla) \bar{H}. \quad (5)$$

Since the density of a fluid particle remains unchanged as we follow it with its motion, we have

$$\varepsilon \frac{\partial \rho}{\partial t} + (\vec{v} \cdot \nabla) \rho = 0. \quad (6)$$

In the equation of motion (2), assuming uniform particle size, spherical shape and small relative velocities between the fluid and particle, the presence of particle adds an extra force term, in the equation of motion (2) proportional to the velocity difference between particles and fluid.

The force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid. Interparticle reactions are ignored for we assume that the distances between the particles are large compared with their diameters. If mN is the mass of the particles per unit volume, then the equations of motions and continuity for the particles are

$$mN \left[\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} \right] = KN(\vec{v} - \vec{u}), \quad (7)$$

$$\frac{\partial N}{\partial t} + \nabla(N\vec{u}) = 0. \quad (8)$$

Let $\delta\rho$, δP , $\vec{v}(u, v, w)$, $\vec{u}(l, r, s)$ and $\vec{h}(h_x, h_y, h_z)$ denote, respectively, the perturbation in the fluid density ρ , the fluid pressure P , the fluid velocity $(0,0,0)$, the particle velocity $(0,0,0)$ and the magnetic field $\vec{H}(H_0(z), 0, 0)$; then the linearized hydromagnetic perturbation equations of Oldroydian viscoelastic fluid in porous medium with suspended particles are

$$\begin{aligned} \frac{\rho}{\varepsilon} \left(1 + \lambda \frac{\partial}{\partial t} \right) \frac{\partial \vec{v}}{\partial t} = & \left(1 + \lambda \frac{\partial}{\partial t} \right) \left[-\nabla \delta P + \vec{g} \delta \rho + \frac{\mu e}{4\pi} \{ (\nabla \times \vec{h}) \times \vec{H} \right. \\ & \left. + (\nabla \times \vec{H}) \times \vec{h} \} + \frac{KN}{\varepsilon} (\vec{u} - \vec{v}) \right] - \frac{\mu}{k_1} \left(1 + \lambda_0 \frac{\partial}{\partial t} \right) \vec{v}, \end{aligned} \quad (9)$$

$$\nabla \cdot \vec{v} = 0, \quad (10)$$

$$\nabla \cdot \vec{h} = 0, \quad (11)$$

$$\varepsilon \frac{\partial \vec{h}}{\partial t} = (\vec{H} \cdot \nabla) \vec{v} - (\vec{v} \cdot \nabla) \vec{H}, \quad (12)$$

$$\varepsilon \frac{\partial}{\partial t} \delta \rho = -w(D\rho), \quad (13)$$

$$\left(\frac{m}{K} \frac{\partial}{\partial t} + 1 \right) \vec{u} = \vec{v}, \quad (14)$$

$$\frac{\partial M}{\partial t} + \nabla \cdot \vec{u} = 0, \quad (15)$$

where:

$$M = \frac{\varepsilon N}{N_0},$$

N_0 and N stand for initial uniform number density and perturbation in number density, and

$$D = \frac{d}{dz}.$$

3. DISPERSION RELATION

Analyzing the perturbation into normal modes, we assume that the perturbation quantities have an x , y and t dependence of the form

$$\exp(ik_x x + ik_y y + nt), \quad (16)$$

where k_x , k_y are the wave numbers along x and y directions, respectively, $k = \sqrt{k_x^2 + k_y^2}$ is the resultant wave number of disturbance and n stands for the growth rate which is, in general, a complex constant. For perturbations of the form (16) equations (9)–(14), after eliminating \vec{u} , give

$$\begin{aligned} & \frac{1}{\varepsilon} \left[\rho + \frac{mN}{\tau n + 1} \right] (1 + \lambda n) nu \\ &= (1 + \lambda n) \left[-ik_x \delta P + \frac{\mu_e}{4\pi} h_z (DH_0) \right] - \frac{\mu}{k_1} (1 + \lambda_0 n) u, \end{aligned} \quad (17)$$

$$\begin{aligned} & \frac{1}{\varepsilon} \left[\rho + \frac{mN}{\tau n + 1} \right] (1 + \lambda n) nv \\ &= (1 + \lambda n) \left[-ik_y \delta P + \frac{\mu_e}{4\pi} H_0 (ik_x h_y - ik_y h_x) \right] - \frac{\mu}{k_1} (1 + \lambda_0 n) v, \end{aligned} \quad (18)$$

$$\begin{aligned} & \frac{1}{\varepsilon} \left[\rho + \frac{mN}{\tau n + 1} \right] (1 + \lambda n) nw \\ &= (1 + \lambda n) \left[-D \delta P - g \delta \rho + \frac{\mu_e}{4\pi} (ik_x h_z H_0 - H_0 Dh_x - h_x DH_0) \right] - \frac{\mu}{k_1} (1 + \lambda_0 n) w, \end{aligned} \quad (19)$$

$$ik_x u + ik_y v + Dw = 0, \quad (20)$$

$$ik_x h_x + ik_y h_y + Dh_z = 0, \quad (21)$$

$$\left. \begin{aligned} \varepsilon n h_x &= ik_x H_0 u - w D H_0, \\ \varepsilon n h_y &= ik_x H_0 v, \\ \varepsilon n h_z &= ik_x H_0 w, \end{aligned} \right\} \quad (22)$$

$$\varepsilon n \delta \rho = -w D \rho, \quad (23)$$

where $\tau = m/K$.

Eliminating δP , $\delta \rho$, u , v , h_x , h_y , h_z between (17)–(19) and using (20)–(23) we obtain

$$\begin{aligned} (1 + \lambda n) & \left[D(\rho Dw) - k^2 \rho w \right] + \frac{gk^2}{n^2} (D\rho)w - \frac{\mu_e}{4\pi n^2} H_0^2 k_x^2 k^2 w \\ & + \frac{\mu_e k_x^2}{4\pi n^2} D(H_0^2 Dw) + \frac{1}{(\tau n + 1)} \{D(mNDw) - k^2 mNw\} \\ & + \frac{\varepsilon}{nk_1} (1 + \lambda_0 n) [D(\mu Dw) - k^2 \mu w] = 0. \end{aligned} \quad (24)$$

4. THE CASE OF EXPONENTIALLY VARYING DENSITY, VISCOSITY, MAGNETIC FIELD AND SUSPENDED PARTICLES

Here we consider the stratification in magnetic field, density, viscosity and suspended particles in the fluid of the depth d as

$$\mu = \mu_0 e^{\beta z}, \quad \rho = \rho_0 e^{\beta z}, \quad H^2 = H_0^2 e^{\beta z} \quad \text{and} \quad N = N_0 e^{\beta z}, \quad (25)$$

where ρ_0 , μ_0 , H_0 , N_0 and β are constants. Equations (25) imply that the coefficient of kinematic viscosity ν and the Alfvén velocity V_A are constant everywhere.

Substituting the values of ρ , μ , H_0^2 , N in (24) and neglecting the effect of heterogeneity on inertia and solving the equation we arrive at

$$(D^2 - k^2)w + \frac{g\beta k^2}{n^2 \left[1 + \frac{(1 + \lambda_0 n) \varepsilon \nu_0}{n(1 + \lambda n) k_1} + \frac{k_x^2 V_A^2}{n^2} + \frac{mN_0 / \rho_0}{(\tau n + 1)} \right]} w = 0, \quad (26)$$

where

$$v_0 = \frac{\mu_0}{\rho_0}, \quad V_A^2 = \frac{\mu_e H_0^2}{4\pi\rho_0} \quad \text{and} \quad \tau = \frac{m}{K}$$

are constants.

Assume that the system is restricted by two planes $z = 0$ and $z = d$ and that both the boundaries are free.

The boundary conditions for the case of two free surfaces are

$$w = D^2 w = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = d. \quad (27)$$

The proper solution of equation (26) satisfying (27) is

$$w = w_0 \sin \frac{m\pi z}{d}, \quad (28)$$

where w_0 is a constant and m is an integer. Substituting (28) in (26) and simplifying we obtain

$$\begin{aligned} & \tau\lambda n^4 + n^3 \left[\tau + \lambda + \frac{\tau\lambda_0 \varepsilon v_0}{k_1} + \frac{\lambda m N_0}{\rho_0} \right] + n^2 \left[1 + \left((\tau + \lambda_0) + \frac{\varepsilon v_0}{k_1} + \frac{m N_0}{\rho_0} \right) \right. \\ & \left. + \tau\lambda \left(k_x^2 V_A^2 - \frac{g\beta k^2}{L} \right) \right] + n \left[\frac{\varepsilon v_0}{k_1} + (\tau + \lambda) \left\{ k_x^2 V_A^2 - \frac{g\beta k^2}{L} \right\} \right] + \left[k_x^2 V_A^2 - \frac{g\beta k^2}{L} \right] = 0, \quad (29) \end{aligned}$$

where

$$L = \frac{m^2 \pi^2}{d^2} + k^2.$$

For the stable stratifications ($\beta < 0$), equation (29) does not have any positive root of n implying thereby that the system is stable for disturbances of all wave numbers.

For the unstable stratifications ($\beta > 0$), the system is stable or unstable depending on whether $k_x^2 V_A^2$ is greater than or less than $g\beta k^2/L$, i.e.,

$$k_x^2 V_A^2 > \frac{g\beta k^2}{L} \quad \text{or} \quad k_x^2 V_A^2 < \frac{g\beta k^2}{L}.$$

The system is clearly unstable for $\beta > 0$ in the absence of a magnetic field and is also unstable if

$$k_x^2 V_A^2 < \frac{g\beta k^2}{L}.$$

However, the system can be completely stabilized by a large magnetic field as can be seen from equation (29) if

$$V_A^2 > \frac{g\beta k^2}{k_x^2 L}.$$

The magnetic field therefore succeeds in stabilizing wave numbers in the range

$$k^2 > \frac{g\beta}{V_A^2} \sec^2 \theta - \left(\frac{m\pi}{d} \right)^2, \quad (30)$$

which are unstable in the absence of magnetic field. Here θ is the angle between k_x and k (i.e., $k_x = k \cos \theta$).

The viscoelasticity, the medium permeability and the suspended particles do not have any qualitative effect on the nature of stability nor instability.

Thus, if

$$\beta > 0 \quad \text{and} \quad k_x^2 V_A^2 < \frac{g\beta k^2}{L}$$

equation (29) has one positive root and one negative root. Let n_0 denote the positive root of (29), then

$$\begin{aligned} & \tau\lambda n_0^4 + n_0^3 \left[\tau + \lambda + \frac{\tau\lambda_0 \varepsilon v_0}{k_1} + \frac{\lambda m N_0}{\rho_0} \right] + n_0^2 \left[1 + (\tau + \lambda_0) + \frac{\varepsilon v_0}{k_1} + \frac{m N_0}{\rho_0} \right. \\ & \left. + \tau\lambda \left(k_x^2 V_A^2 - \frac{g\beta k^2}{L} \right) \right] + n_0 \left[\frac{\varepsilon v_0}{k_1} + (\tau + \lambda) \left\{ k_x^2 V_A^2 - \frac{g\beta k^2}{L} \right\} \right] + \left[k_x^2 V_A^2 - \frac{g\beta k^2}{L} \right] = 0. \quad (31) \end{aligned}$$

To study the behaviour of the growth rate of unstable modes with respect to kinematic viscosity, medium permeability, stress relaxation time and strain retardation time and particle number density, we examine the natures of $\frac{dn_0}{dv_0}$, $\frac{dn_0}{dk_1}$, $\frac{dn_0}{d\lambda}$, $\frac{dn_0}{d\lambda_0}$

and $\frac{dn_0}{dN_0}$ analytically. Equation (31) yields

$$\begin{aligned} \frac{dn_0}{dk_1} = & \frac{\frac{\varepsilon n_0 v_0}{k_1^2} (n_0^2 \tau \lambda_0 + 1)}{4n_0^3 \tau \lambda + 3n_0^2 \left[\tau + \lambda + \frac{\tau\lambda_0 \varepsilon v_0}{k_1} + \frac{\lambda m N_0}{\rho_0} \right] + 2n_0 \left[1 + (\tau + \lambda_0) \frac{\varepsilon v_0}{k_1} \right.} \\ & \left. + \frac{m N_0}{\rho_0} + \tau\lambda \left(k_x^2 V_A^2 - \frac{g\beta k^2}{L} \right) \right] + \left[\frac{\varepsilon v_0}{k_1} + (\tau + \lambda) \left\{ k_x^2 V_A^2 - \frac{g\beta k^2}{L} \right\} \right]}, \quad (32) \end{aligned}$$

$$\frac{dn_0}{dN_0} = \frac{-\frac{n_0^2 m}{\rho_0}(\lambda n_0 + 1)}{4n_0^3 \tau \lambda + 3n_0^2 \left[\tau + \lambda + \frac{\tau \lambda_0 \varepsilon v_0}{k_1} + \frac{\lambda m N_0}{\rho_0} \right] + 2n_0 \left[1 + (\tau + \lambda_0) \frac{\varepsilon v_0}{k_1} \right.}$$

$$\left. + \frac{m N_0}{\rho_0} + \tau \lambda \left(k_x^2 V_A^2 - \frac{g \beta k^2}{L} \right) \right] + \left[\frac{\varepsilon v_0}{k_1} + (\tau + \lambda) \left\{ k_x^2 V_A^2 - \frac{g \beta k^2}{L} \right\} \right]}, \quad (33)$$

$$\frac{dn_0}{dv_0} = \frac{-\frac{n_0 \varepsilon}{k_1} (n_0^2 \tau \lambda_0 + 1)}{4n_0^3 \tau \lambda + 3n_0^2 \left[\tau + \lambda + \frac{\tau \lambda_0 \varepsilon v_0}{k_1} + \frac{\lambda m N_0}{\rho_0} \right] + 2n_0 \left[1 + (\tau + \lambda_0) \frac{\varepsilon v_0}{k_1} \right.}$$

$$\left. + \frac{m N_0}{\rho_0} + \tau \lambda \left(k_x^2 V_A^2 - \frac{g \beta k^2}{L} \right) \right] + \left[\frac{\varepsilon v_0}{k_1} + (\tau + \lambda) \left\{ k_x^2 V_A^2 - \frac{g \beta k^2}{L} \right\} \right]}, \quad (34)$$

$$\frac{dn_0}{d\lambda} = \frac{-n_0 \left[\tau n_0^3 + n_0^2 \left(1 + \frac{m N_0}{\rho_0} \right) + (n_0 \tau + 1) \left(k_x^2 V_A^2 - \frac{g \beta k^2}{L} \right) \right]}{4n_0^3 \tau \lambda + 3n_0^2 \left[\tau + \lambda + \frac{\tau \lambda_0 \varepsilon v_0}{k_1} + \frac{\lambda m N_0}{\rho_0} \right] + 2n_0 \left[1 + (\tau + \lambda_0) \frac{\varepsilon v_0}{k_1} \right.}$$

$$\left. + \frac{m N_0}{\rho_0} + \tau \lambda \left(k_x^2 V_A^2 - \frac{g \beta k^2}{L} \right) \right] + \left[\frac{\varepsilon v_0}{k_1} + (\tau + \lambda) \left\{ k_x^2 V_A^2 - \frac{g \beta k^2}{L} \right\} \right]}, \quad (35)$$

$$\frac{dn_0}{d\lambda_0} = \frac{(1 + \tau n_0) n_0^2 \frac{\varepsilon v_0}{k_1}}{4n_0^3 \tau \lambda + 3n_0^2 \left[\tau + \lambda + \frac{\tau \lambda_0 \varepsilon v_0}{k_1} + \frac{\lambda m N_0}{\rho_0} \right] + 2n_0 \left[1 + (\tau + \lambda_0) \frac{\varepsilon v_0}{k_1} \right.}$$

$$\left. + \frac{m N_0}{\rho_0} + \tau \lambda \left(k_x^2 V_A^2 - \frac{g \beta k^2}{L} \right) \right] + \left[\frac{\varepsilon v_0}{k_1} + (\tau + \lambda) \left\{ k_x^2 V_A^2 - \frac{g \beta k^2}{L} \right\} \right]}. \quad (36)$$

Let

$$\frac{1}{\tau + \lambda + 2n_0\tau\lambda} \left[4n_0^3\tau\lambda + 3n_0^2 \left(\tau + \lambda + \frac{\tau\lambda_0\varepsilon v_0}{k_1} + \frac{\lambda m N_0}{\rho_0} \right) + 2n_0 \left(1 + (\tau + \lambda_0) \frac{\varepsilon v_0}{k_1} + \frac{m N_0}{\rho_0} \right) + \frac{\varepsilon v_0}{k_1} \right] = R$$

and

$$\frac{n_0^2}{(1 + \tau n_0)} \left[\tau n_0 + 1 + \frac{m N_0}{\rho_0} \right] = S.$$

It is evident from (35) that either

- (i) $\left| k_x^2 V_A^2 - \frac{g\beta k^2}{L} \right| < R$ and S or $\left| k_x^2 V_A^2 - \frac{g\beta k^2}{L} \right| > R$ and S , then $\frac{dn_0}{d\lambda}$ is negative.
- (ii) $S > \left| k_x^2 V_A^2 - \frac{g\beta k^2}{L} \right| > R$, then $\frac{dn_0}{d\lambda}$ is positive.

Thus, the growth rates decrease or increase with an increase in stress relaxation time. Also, it is evident from (32), (33), (34) and (36) that if

$$\left| k_x^2 V_A^2 - \frac{g\beta k^2}{L} \right| < R \quad \text{or} \quad \left| k_x^2 V_A^2 - \frac{g\beta k^2}{L} \right| > R,$$

then

$$\frac{dn_0}{dN_0} \quad \text{and} \quad \frac{dn_0}{dv_0}$$

are always negative or positive and

$$\frac{dn_0}{dk_1} \quad \text{and} \quad \frac{dn_0}{d\lambda_0}$$

are always positive or negative.

The growth rates, therefore, decrease or increase with an increase in particle density and kinematic viscosity; similarly, growth rates increase or decrease with an increase in medium permeability and strain retardation time.

Thus, the growth rates both increase (for certain wave numbers) and decrease (for different wave numbers) with an increase in kinematic viscosity, medium permeability, suspended particles number, density, stress relaxation time and strain retardation time.

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