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# EFFECTS OF NEUTRAL GAS FRICTION ON THE INSTABILITY OF SUPERPOSED STREAMING PLASMAS

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**Abstract:** The instability of plane interface separating two superposed partially ionized streaming plasmas in a uniform vertical magnetic field in the presence of effect of surface tension is discussed. The dispersion relation has been derived by using the normal mode technique and solved numerically. It has been found that the collision frequency, streaming velocity and ion viscosity have destabilizing influence while surface tension has stabilizing influence on the growth rate of unstable mode of disturbance.

## 1. INTRODUCTION

The problem of instability of a plane interface between two superposed fluids which are in relative horizontal motion parallel to their interface is known as the Kelvin–Helmholtz instability. The Kelvin–Helmholtz instability arises when air is blown over mercury or when a highly ionized hot plasma is surrounded by a slightly cold ionized gas. From geophysical point of view, this situation arises when a meteor enters the Earth's atmosphere. CHANDRASEKHAR [5] has given a detailed account of the various investigations of this problem as investigated by different researchers for incompressible fluids. For the case of a low – $\beta$ magnetized plasma in the presence of effect of compressibility, D'ANGELO [6] found that a velocity difference between adjacent field-aligned flows, on the order of the ion – acoustic speed $C_s$, is sufficient for instability. GERWIN [9] has examined the stability problem of the non-conducting streaming gas flowing over an incompressible conducting fluid. KALRA et al. [10] have investigated Kelvin–Helmholtz instability of superposed fluids in the presence of the effect of magnetic resistivity, while BHATIA and STEINER [4] have analysed this problem for partially ionized plasma.

The Kelvin–Helmholtz instability of superposed plasmas has been considered by SHIVAMOGGI [12], who has shown that in the case of plasma immersed in a horizontal magnetic field, the inclusion of finite resistivity leads to new unstable modes. However, in the later stages of the derivation of the dispersion relation, he has assumed that the densities of two plasmas are equal. ELDABE and HASSAN [8] have investigated the effects of magnetic field and surface tension on the Kelvin–Helmholtz instability of superposed plasma with finite resistivity.

The K–H instability problem of superposed fluids has attracted the attention of researchers in recent years under different assumptions. D'ANGELO and SONG [7] have considered the problem of dusty plasma while MALICK and SINGH [11] have dealt with the problem of chaos in K–H instability in magnetic fluids by using the bifurcation method and applying Melnikov function more recently BENJAMIN and BRIDGES [1] have derived the Hamiltonian formulation of the K–H instability of superposed fluids and obtained the result for both linear and non-linear instability study for viscous fluids. The K–H instability of two superposed viscous conducting fluids in a uniform vertical magnetic field is discussed in the presence of effect of surface tension and permeability of porous medium by BHATIA and SHARMA [3].

In cosmic physics there are several situations such as in chromosphere, solar photosphere and in cool interstellar clouds where the plasmas are not frequently fully ionized but may instead be partially ionized so that interaction between the ionized fluid and neutral gas becomes important. BHATIA and MATHUR [2] have discussed the linear Kelvin–Helmholtz discontinuity in partially ionized plasmas in a uniform vertical magnetic field.

It would, therefore, be of interest to examine the K–H instability problem in superposed partially ionized plasmas in the presence of effects of ion viscosity and surface tension. This aspect forms the basis of this paper wherein the effect of neutral gas friction on the stability of superposed streaming plasmas is studied for the longitudinal mode of propagation.

## 2. LINEARIZED PERTURBATION EQUATIONS

We consider the motion of the mixture of an incompressible, infinitely conducting viscous fluid of density $\rho$ and neutral gas of density $\rho_d$ ($\rho_d < \rho$) having a streaming velocity $\vec{V}$ along the horizontal direction. The magnetic field is taken to be uniform and acting along a vertical direction. Let us make the following assumptions.

(1) The magnetic field interacts with the conducting fluids.
(2) The steady state velocities of the two components (ionized fluid and neutral gas) are equal.
(3) The effects of the pressure gradient; gravity are ignored on the neutrals.
(4) Both the components behave like continuums.

Retaining only the linear terms in the perturbed quantities in the governing equations and making use of the above assumptions, we get the linearized perturbation equations

$$\rho\frac{\partial\vec{u}}{\partial t}+\rho(\vec{V}.\nabla)=-\nabla\delta p+(\nabla\times\vec{h})\times\vec{H}+\vec{g}\delta\rho+\mu\nabla^2\vec{u}$$

$$+\rho_d \upsilon_c (\vec{u}_d - \vec{u}) + \sum_s \left[ T_s \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \delta z_s \right] \delta(z - z_s) , \tag{1}$$

$$\frac{\partial}{\partial t} \vec{u}_d + (\vec{V}.\nabla)\vec{u}_d = -\upsilon_c (\vec{u}_d - \vec{u}) , \tag{2}$$

$$\frac{\partial}{\partial t}(\delta\rho) + (\vec{V}.\nabla)\delta\rho + (\vec{u}.\nabla)\rho = 0 , \tag{3}$$

$$\frac{\partial}{\partial t}\vec{h} + (\vec{V}.\nabla)\vec{h} = (\vec{H}.\nabla)\vec{u} , \tag{4}$$

$$\nabla.\vec{h} = 0 , \tag{5}$$

$$\nabla.\vec{u} = 0 , \tag{6}$$

where $\vec{u}$ = ($u$, $v$, $w$), $\vec{h}$ = ($h_x$, $h_y$, $h_z$), $\delta\rho$ and $\delta p$ are respectively the perturbation in velocity, magnetic field $\vec{H}$ , density $\rho$ and pressure $p$ of the ionized fluid, while $\vec{u}_d$ denotes the velocity of neutral gas. Here, $T$, $\mu$, $\vec{g}$ = (0, 0, $-g$) and $\upsilon_c$ denote surface tension, coefficient of viscosity, acceleration due to gravity and collision frequency between the two components of the plasma. In equation (1) $\delta(z - z_s)$ denote Dirac's delta function.

We take streaming velocity $\vec{V}$ = ($U$, 0, 0) and magnetic field $\vec{H}$ = (0, 0, $H$).

For longitudinal wave propagations we analyse the disturbance in terms of normal modes and dependence on space coordinates $x$, $z$ and time $t$ is of the form

$$F(z) \exp (ik_x x + nt) \tag{7}$$

where $F(z)$ is some function of $z$ and $k_x$ is the wave number along the $x$-direction and $n$, may be complex, denotes the rate at which system deviates from the equilibrium.

Using equation (7) in (1) to (6), we get

$$\rho n_c u = -ik_x \delta p + H(Dh_x - ik_x h_z) + \mu(D^2 - k_x^2)u , \tag{8}$$

$$\rho n_c v = HDh_y + \mu(D^2 - k_x^2)v , \tag{9}$$

$$\rho n_c w = -D\delta p - g\delta\rho + \mu(D^2 - k_x^2)w - \frac{k_x^2 T}{n}\delta(z - z_s)w , \tag{10}$$

$$(n + ik_x U)\delta\rho = -(D\rho)w , \tag{11}$$

$$(n + ik_x U)h_x = HDu , \tag{12}$$

$$(n+ik_xU)h_y = HDv\,, \tag{13}$$

$$(n+ik_xU)h_y = HDw\,, \tag{14}$$

$$ik_xh_x = -Dh_z\,, \tag{15}$$

$$ik_xu = -Dw\,, \tag{16}$$

where

$$D = \frac{d}{dz}\,, \tag{17}$$

$$n_c = (n+ik_xu)\left[1+\frac{\beta\upsilon_c}{n+ik_xU+\upsilon_c}\right], \tag{18}$$

and

$$\beta = \frac{\rho_d}{\rho}\,. \tag{19}$$

Eliminating some of the variables from the above equations we finally obtain an equation in $w$

$$\frac{H^2(D^2-k_x^2)D^2w}{n'} - \rho n_c(D^2-k_x^2)w$$
$$-\frac{g(D\rho)k_x^2w}{n'} + \frac{k_x^4T}{n'}\delta(z-z_s)w + \mu(D^2-k_x^2)^2w = 0 \tag{20}$$

where

$$n' = n + ik_xU. \tag{21}$$

## 3. TWO SUPERPOSED STREAMING PLASMAS

We now consider the case when two superposed plasmas of uniform densities $\rho_1$ and $\rho_2$, uniform viscosities $\mu_1$ and $\mu_2$, magnetic fields $H_1$ and $H_2$ and with streaming velocities $U_1$ and $U_2$ are separated by a horizontal boundary $z = 0$. Then in both regions of constant density, equation (20) becomes

$$(D^2-k_x^2)(D^2-M^2)w = 0 \tag{22}$$

where

$$M^2 = \left(k_x^2+\frac{n_c}{\upsilon}\right)\left(1+\frac{V_A^2}{n'\upsilon}\right)^{-1}. \tag{23}$$

Here,

$$\upsilon = \frac{\mu}{\rho} \quad \text{and} \quad V_A^2 = \frac{H^2}{\rho} \tag{24}$$

are the kinematic viscosity and Alfven velocity, respectively.

Now, seeking the solution of equation (22) which remains bounded in the two regions, we obtain

$$w_1 = A_1 n_{c1} e^{k_x z} + B_1 n_{c1} e^{M_1 z}, \quad z < 0, \tag{25}$$

$$w_2 = A_2 n_{c2} e^{-k_x z} + B_2 n_{c2} e^{-M_2 z}, \quad z > 0, \tag{26}$$

where $A_1$, $B_1$, $A_2$ and $B_2$ are constants and $M_1$ and $M_2$ are positive square roots of equation (23) for the two regions, and

$$n_{c1} = n_1' \left( 1 + \frac{\beta \upsilon_c}{n_1' + \upsilon_c} \right), \tag{27}$$

$$n_{c2} = n_2' \left( 1 + \frac{\beta \upsilon_c}{n_2' + \upsilon_c} \right), \tag{28}$$

$$n_1' = n + ik_x U_1, \tag{29}$$

$$n_2' = n + ik_x U_2. \tag{30}$$

The above solutions must satisfy certain boundary conditions. These boundary conditions require that at the interface $z = 0$,

$$w, \quad Dw \quad \text{and} \quad \mu(D^2 + k_x^2)w \tag{31}$$

must be continuous.

If we integrate equation (20) across the interface $z = 0$, we obtain another condition

$$\begin{aligned} &\left[ n_{c2}\rho_2 Dw_2 - \mu_2 (D^2 - k_x^2) Dw_2 + \frac{H^2}{n_2'} (D^2 - k_x^2) Dw_2 \right]_{z=0} \\ &- \left[ n_{c1}\rho_1 Dw_1 - \mu_1 (D^2 - k_x^2) Dw_1 + \frac{H^2}{n_1'} (D^2 - k_x^2) Dw_1 \right]_{z=0} \\ &= = -k_x^2 \left[ g \left( \frac{\rho_2}{n_2'} - \frac{\rho_1}{n_1'} \right) - k_x^2 T \right] w_0 - 2k_x^2 (\mu_2 - \mu_1)(Dw)_0 \end{aligned} \tag{32}$$

where $w_0$ and $(Dw)_0$ are unique values of their quantities at $z = 0$, and applying conditions (31) and (32) to solutions (25) and (26), we get

$$A_1 + B_1 = A_2 + B_2 \,, \tag{33}$$

$$k_x A_1 + M_1 B_1 = -k_x A_2 - M_2 B_2, \tag{34}$$

$$\mu_1[2k_x^2 A_1 + (M_1^2 + k_x^2)B_1] = \mu_2[2k_x^2 A_2 + (M_2^2 + k_x^2)B_2] \tag{35}$$

$$\begin{aligned} &-k_x\rho_2 n_{c2} A_2 - \frac{H^2 k_x^2}{n_2'} M_2 B_2 - k_x \rho_1 n_{c1} A_1 - \frac{H^2 k_x^2}{n_1'} M_1 B_1 \\ &= \frac{g k_x^2}{2}\left(\frac{\rho_2}{n_2'} - \frac{\rho_1}{n_1'}\right)(A_1 + B_1 + A_2 + B_2) \\ &- k_x^2(\mu_2 - \mu_1)(k_x A_1 + M_1 B_1 - k_x A_2 - M_2 B_2) + \frac{k_x^4}{2n^2} T(A_1 + B_1 + A_2 B_2)\,. \end{aligned} \tag{36}$$

Eliminating the constants from equations (33) to (36), we obtain

$$\begin{vmatrix} 1 & 1 & -1 & -1 \\ k_x & M_1 & k_x & M_2 \\ 2\mu_1 k_x^2 & \mu_1(M_1^2 + k_x^2) & -2\mu_2 k_x^2 & -\mu_2(M_2^2 + k_x^2) \\ -n_{c1}\alpha_1 + \dfrac{R}{2} - C + S & \dfrac{R}{2} - \dfrac{M_1 C}{k_x} + S & -n_{c2}\alpha_2 + \dfrac{R}{2} + C + S & \dfrac{R}{2} + \dfrac{M_2 C}{k_x} + S \end{vmatrix} = 0 \tag{37}$$

where

$$\alpha_{1,2} = \frac{\rho_{1,2}}{\rho_1 + \rho_2}\,, \tag{38}$$

$$R = g k_x^2 \left(\frac{\alpha_1}{n_1'} - \frac{\alpha_2}{n_2'}\right), \tag{39}$$

$$S = \frac{k_x^4 T}{2}\left(\frac{1}{n_2'} - \frac{1}{n_1'}\right), \tag{40}$$

$$C = \frac{k_x^2}{(\rho_1 + \rho_2)}(\mu_1 - \mu_2) = k_x^2(\alpha_1 \upsilon_1 - \alpha_2 \upsilon_2)\,. \tag{41}$$

On evaluating the determinant (37), we obtain a characteristic equation

$$(M_1-k_x)\left[2k_x^2(\alpha_1\upsilon_1-\alpha_2\upsilon_2)\left\{n_{c2}\alpha_2+\frac{C}{k_x}(M_2-k_x)\right\}\right.$$

$$\left.+\alpha_2\upsilon_2(M_2^2-k_x^2)(R-n_{c1}\alpha_1-n_{c2}\alpha_2)\right]-2k_x\left[(\alpha_1\upsilon_1(M_1^2-k_x^2)\left\{n_{c2}\alpha_2+\frac{C}{k_x}(M_2-k_x)\right\}\right.$$

$$\left.+\alpha_2\upsilon_2(M_2^2-k_x^2)\left\{n_{c1}\alpha_1-\frac{C}{k_x}(M_1-k_x)\right\}\right]+(M_2-k_x)\left[\alpha_1\upsilon_1(M_1^2-k_x^2)\right.$$

$$\left.(R-n_{c1}\alpha_1-n_{c2}\alpha_2)-2k_x^2(\alpha_1\upsilon_1-\alpha_2\upsilon_2)\left\{n_{c1}\alpha_1-\frac{C}{k_x}(M_1-k_x)\right\}\right]=0\,. \tag{42}$$

The dispersion relation (42) is quite complex, particularly as $M_1$ and $M_2$ involve square roots, we therefore, carry out the instability for the case of highly viscous superposed plasma for thus we can write

$$M_1=k_x+\frac{n_{c1}}{2\upsilon_1k_x}-\frac{V_{A1}^2k_x}{\upsilon_1n_1'}\,, \tag{43}$$

$$M_2=k_x+\frac{n_{c2}}{2\upsilon_2k_x}-\frac{V_{A2}^2k_x}{\upsilon_2n_2'}\,. \tag{44}$$

Neglecting square and higher order terms of $\frac{1}{\upsilon_{1,2}}$, i.e., assuming that both plasmas are highly viscous, substituting the values of $M_1$ and $M_2$ from (43) and (44) in equation (42) and substituting the values of $R$, $S$ and $C$ from (39) to (41) and setting

$$U_1=U_2=U;\quad V_1=V_2=V;\quad \upsilon_1=\upsilon_2=\upsilon, \tag{45}$$

and on writing

$$n^*=\frac{n}{\sqrt{g}}\,,\quad U^*=\frac{U}{\sqrt{g}}\,,\quad V_A{}^*=V_A^*=\frac{V_A}{\sqrt{g}}\,,\quad \upsilon^*=\frac{\upsilon}{\sqrt{g}}\,,\quad \upsilon_C{}^*=\frac{\upsilon_c}{\sqrt{g}}\,, \tag{46}$$

we obtain the dispersion relation

$$\sum_{i=0}^{11}A_in^{*i}=0 \tag{47}$$

where $A_i$'s are complex and involve $U_{1,2}$, $\nu_{1,2}$, $\nu_{c1,2}$, $V_{1,2}$ and $\alpha_{1,2}$. The coefficients are not given here as they are very lengthy expressions.

## 4. DISCUSSION

It is not easy to obtain analytical result from equation (47) as it is quite complex. We have, therefore, solved it numerically for different values of physical parameters involved.

For numerical estimation of the roots $n$ from equation (47), we set $\alpha_1 = 0.25$, $\alpha_2 = 0.75$ (unstable configuration), $V = 1.0$, $\beta = 0.1$. The non-dimensionalised equation (47) has been solved numerically for several values of the parameters $U$, $\nu_c$ and $T$ for fixed values of other parameters to examine the dependence on $n$ of $U$, $\nu_c$ and $T$.

These calculations are presented in Figs. 1–3, where we have plotted the growth rate $n$ against wave number $k_x$ for several values of parameters characterizing streaming velocity, ion viscosity and collision frequency and surface tension.

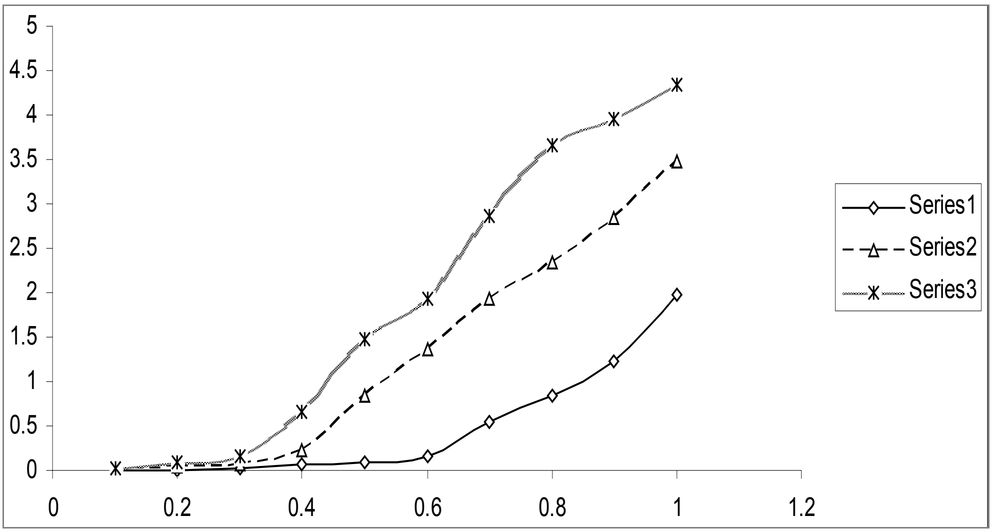


Fig. 1. Plot of growth rate $n$ against wave number $k_x$ for $U = 5, 10, 15$, taking $\alpha_1 = 0.25$, $\alpha_2 = 0.75$, $\nu = 2$, $\nu_c = .1$ $V = 1$ and $T = 0.1$. Series 1, 2, 3 stand for $U = 5, 10, 15$, respectively

It is seen from Fig. 1 that growth rate increases as streaming velocity increases for the same $k_x$, showing thereby destabilizing influence. It is also seen from the same figure that growth rate also increases as collision frequency increases for the same $k_x$, showing thereby destabilizing influence, while Fig. 3 shows that the growth rate decreases as surface tension increases for the same $k_x$, showing thereby stabilizing influence of surface tension.

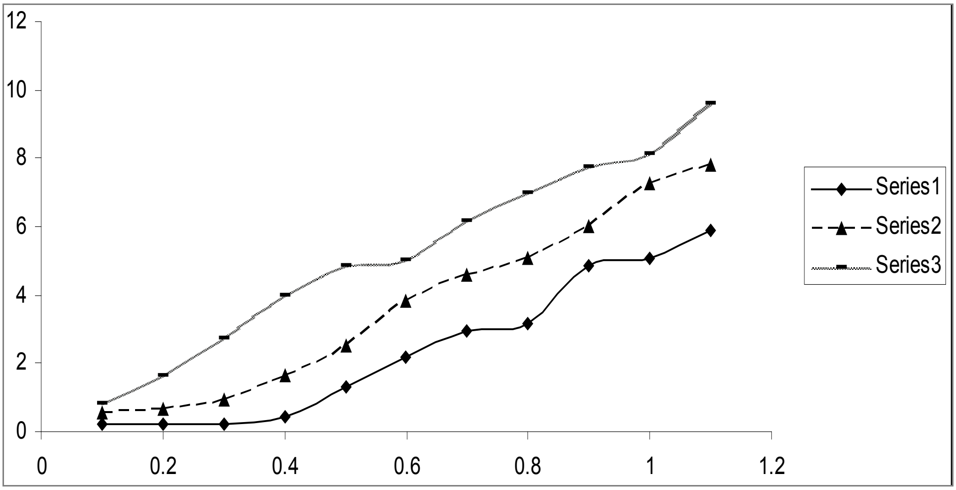


Fig. 2. Plot of growth rate $n$ against wave number $k_x$ for $\nu_c = 0.1, 0.2, 0.3$,
taking $\alpha_1 = 0.25$, $\alpha_2 = 0.75$, $\nu = 2$, $U = 5$, $V = 1$ and $T = 0.1$.
Series 1, 2, 3 stand for $\nu_c = 0.1, 0.2, 0.3$, respectively

We may thus conclude that in the presence of streaming motion, the collision frequency has destabilizing influence while surface tension shows stabilizing influence. Here, in the present case streaming motion also has destablizing influence on the unstable configuration.

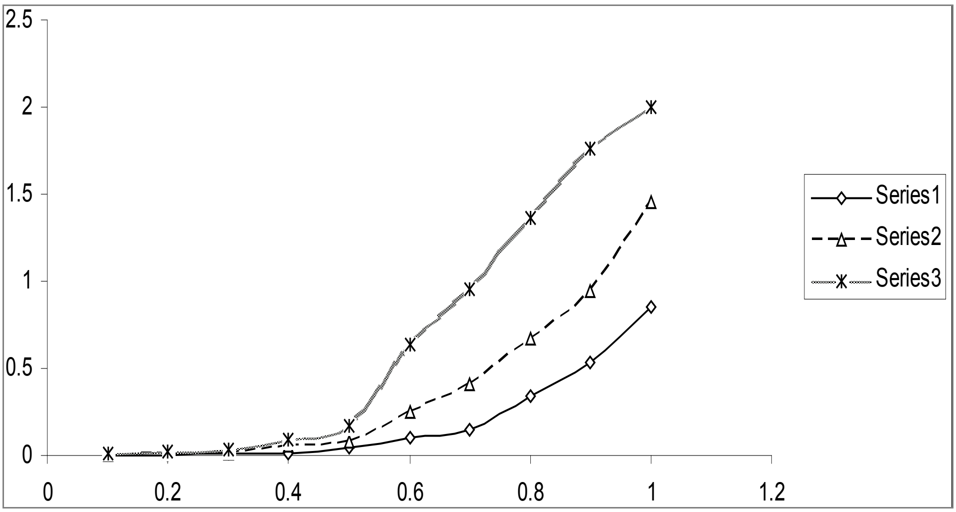


Fig. 3. Plot of growth rate $n$ against wave number $k_x$ for $T = 0.1, 0.2, 0.3$,
taking $\alpha_1 = 0.25$, $\alpha_2 = 0.75$, $\nu = 2$, $U = 5$, $V = 1$ and $\nu_c = 0.1$.
Series 1, 2, 3 stand for $T = 0.3, 0.2, 0.1$, respectively

## REFERENCES


[1] BENJAMIN T.B., BRIDGES T.J., J. Fluid Mech., 1997, 333, 1.
[2] BHATIA P.K., MATHUR R.P., Indian J. Phy., 2004, 78(10), 1147.
[3] BHATIA P.K., SHARMA A., Proc. Nat. Acad. Sci., 2003, 73(A), 497.
[4] BHATIA P.K., STEINER, J.M., Aust. J. Phys., 1974, 27, 53.
[5] CHANDRASEKHAR S., *Hydrodynamic and hydromagnetic Stability*, Clarendon Press, Oxford 1961.
[6] D'ANGELO N., Phys. Fluids, 1965, 8, 1748.
[7] D'ANGELO N., SONG B., Planetary and Space Sci., 1990, 38, 1577.
[8] ELDABE N.T., HASSAN A.A.A., Bull. Cal. Math. Soc., 1991, 83, 407.
[9] GERWIN R.A., Rev. Mod. Phy., 1968, 40, 652.
[10] KALRA G.L., KATHURIA S.N., HOSKING R.J., LISTER G.G., J. Plasma Phys., 1970, 4, 451.
[11] MALIK S.K., SINGH M., Phys. Fluids, A, 1992, 4(12), 2915.
[12] SHIVAMOGGI B.K., Physica Scripta, 1981, 24, 49.