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TECHNICAL NOTE

COMPARISON BETWEEN TWO METHODS FOR ESTIMATING THE VERTICAL SCALE OF FLUCTUATION FOR MODELING RANDOM GEOTECHNICAL PROBLEMS

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Abstract: The design process in geotechnical engineering requires the most accurate mapping of soil. The difficulty lies in the spatial variability of soil parameters, which has been a site of investigation of many researches for many years. This study analyses the soil-modeling problem by suggesting two effective methods of acquiring information for modeling that consists of variability from cone penetration test (CPT). The first method has been used in geotechnical engineering, but the second one has not been associated with geotechnics so far. Both methods are applied to a case study in which the parameters of changes are estimated. The knowledge of the variability of parameters allows in a long term more effective estimation, for example, bearing capacity probability of failure.

Key words: *scale of fluctuation, spatial variability*

1. INTRODUCTION

The randomness seems to be one of the most disturbing issues around us and it is particularly noticeable in the case of soil. Unfortunately, popular computational methods continue to treat soil as a homogenized structure with uniform parameters. However, geotechnical engineers investigate the real behavior of geomaterials and describe it by the parameters useful for modeling. The pioneering works on this subject were published by Lumb (1966, 1970) and Schultze (1972). The concepts of modeling the soil parameters change with the increase of computational ability. One of them is to applicate a random function and random field in the modeling of soil parameters (Lumb 1975, Alonso and Krisek 1975, Vanmarcke 1977a, 1977b, 1983). This problem remains relevant in the literature (e.g., Vessia et al. 2011, Stuedlein et al. 2012, Cao and Wang 2014, Lloret-Cabot et al. 2014). This paper includes a type of heterogeneity attributed to spatial soil variability and defines the vertical variation of soil properties from one point to another. The aim of this study is to consider different methods for the estimating inherent variability from the typical CPT tests. The values obtained from CPT, as q_c – cone resistance, f_s – friction resistance and u – pore pressure allow us to describe: soil layer, local inclusions that could affect the average value and the trend as well as variability of the ground within the homogenized layer (Jaksa et al. [10]). In general, methods of estimating the soil spatial variability assuming that the soil structure $q(z)$ are described by the trend $t(z)$ and the residual fluctuation $x(z)$.

$$q(z) = t(z) - x(z) \ . \tag{1}$$

If the residual value of the measured soil is stationary, at least in a weak sense, the random variable may be described by standard deviation and autocorrelation function which make it one direction random field. The value resulting directly from the assumption of the autocorrelation function is the scale of fluctuation θ, (the distance between the correlated points field, Vanmarcke [24]).

$$\theta = \lim_{t\to\infty} T\gamma(T) \tag{2}$$

if the limit exists, the scale of fluctuation might be describe as

$$\theta = \frac{2}{\sigma^2}\int_0^\infty C(\tau)d\tau \tag{3}$$

where $C(\tau)$ is the covariance function, σ^2 is the variance and $\rho(\tau)$ is the correlation coefficient.

$$C(\tau) = \sigma^2 \rho(\tau) \tag{4}$$

Little correlation will be shown between points separated by a distance father than θ. However, the distant points in the range of less than θ are strongly correlated. Knowing this value, it is possible to determine more realistic behavior of geotechnical structures using advanced probabilistic methods (e.g., Fenton and Griffiths 2008).

The starting point which leads to determine the scale of the fluctuations θ is de-trending $t(z)$ of the values obtained as a result of the test CPT (Vessia et al. 2011, Lloret-Cabot et al. 2014). Then, the part of the residual requires stationary verification in a weak sense, which allows to determine whether:

- the standard deviation is constant with depth,
- the correlation depends only on the distance between observations (only on the scale of fluctuation).

The stationary verification may be performed by dividing part of $x(z)$ by the standard deviation determined from that part (σ_{RES}). If the normalized de-trended cone resistance (q_c^{TZ}) gives zero mean value ($\mu_R = 0$) and unit standard deviation ($\sigma_R = 1$), it is possible to generate normal random field. Conclusion – obtained random field is stationary. The de-trend cone resistance value (q_c^{TZ}), the standard deviation (σ_R) and the mean value (μ_R) are necessary to estimate the empirical correlation function $\hat{\rho}(\tau)$

$$\hat{\rho}(\tau_j) = \frac{1}{\sigma_R^2 (k-j)} \sum_{i=1}^{k-j+1} (X_i - \mu_R)(X_{i+1} - \mu_R), \tag{5}$$

where X_i is the estimated de-trend cone resistance value (q_c^{TZ}), k is the number of observations in the study, τ_j corresponds to the depth of CPT as a multiplication the number of observations $j = 1, 2, ..., k$, and adopted step j $\Delta\tau$, $\tau_j = j\Delta\tau$.

To estimate the scale of fluctuation a theoretical correlation model should be assumed. Many models are described in literature. Some of most popular positive definite functions are shown in Table 1.

Table 1. Positively defined correlation functions of one variable

Correlation model	Expression
Gaussian	$\rho(\tau) = \exp\left\{-\pi\left(\frac{\lvert\tau\rvert}{\theta}\right)^2\right\}$
Markov	$\rho(\tau) = \exp\left\{\frac{-2\lvert\tau\rvert}{\theta}\right\}$

Vanmarcke in 1977 suggested a method of estimation of the scale of fluctuation as a comparison of the theoretical and empirical correlation models. The method is considered the foundation and it is widely employed (Campanella, Wickremesinghe, and Robertson 1987; DeGroot and Baecher 1993; Fenton 1999; Baecher and Christian 2003; Wackernagel 2003; Uzielli, Vannucchi, and Phoon 2005; Fenton and Griffiths 2008, M. Lloret-Cabot et al. 2014).

$$\sum_{j=1}^{k} \tau_j (\hat{\rho}(\tau_j) - \rho(\tau_j))\rho(\tau_j) = 0 \tag{6}$$

where $\rho(\tau)$ is the theoretical correlation function (from Table 1), $\hat{\rho}(\tau)$ is the empirical correlation function.

The method based on variance reduction functions both as a parameter of evaluation of the size of the data and as a theoretical model. (Vanmarcke 1984 and Wickremesinghe and Camapanella 1993, Jaksa, Kaggwa, and Brooker, 1993; Hicks and Onisiphorou, 2005; Lloret, Hicks, and Wong 2012; Lloret-Cabot, Hicks, and Nuttall 2013).

The alternative concept of estimating the scale of fluctuation is a method derived from signal theory and has not been associated with geotechnics so far. It is possible to apply this method with a large number of measurements. The scale of the fluctuation θ is related to the average distance $\bar{d}$, which represents the section of value different than the CPT trend. The concept is the result of the Rice's formula (Rice 1944) and it defines an average frequency of the average value exceedances by a random function as $\frac{1}{\bar{d}}$. Approximately:

$$\frac{1}{\bar{d}} \approx \frac{1}{\pi} \sqrt{\left. \frac{\partial^2 \rho(\tau)}{\partial \tau^2} \right|_{\tau=0}} \tag{7}$$

Table 2. Rice equations for scale of fluctuation

Correlation model	Scale of fluctuation
Gaussian	$\theta = \bar{d}\sqrt{\frac{2}{\pi}}$
Markov	$\theta = \frac{2\bar{d}}{\pi}$

According to the Rice method, the first thing that must be determined after the de-trend of the CPT measurements is the length of sections of the same sign. Then, the average of the lengths $\bar{d}$ needs to be appointed. At the same time in accordance with Rice's formula the theoretical correlation function is substituted and relationship between the average length of the sections and the scale of fluctuation is estimated. Equations of the scale of fluctuation estimated for the functions (has been shown in Table 1) are described in Table 2.

2. RESULTS

This paper investigates the data from CPT measurements in Świebodzice (Bagińska et al., 2012). In particular, tests of the embankment construction with thickness 5.5 m were demonstrated. Total length of the test equaled 7.2 m. The embankment was made in the vast majority of cohesive soils. Fig. 1 shows the course of CPT measurements with the separation of the different layers of geotechnical graph of q_c, f_s and u.

First, the trends of cone tip resistance were determined. Fig. 2a shows matching the linear trend with total CPT. Fig. 2b shows matching the quadratic trend with total CPT. There were also separate analyses of

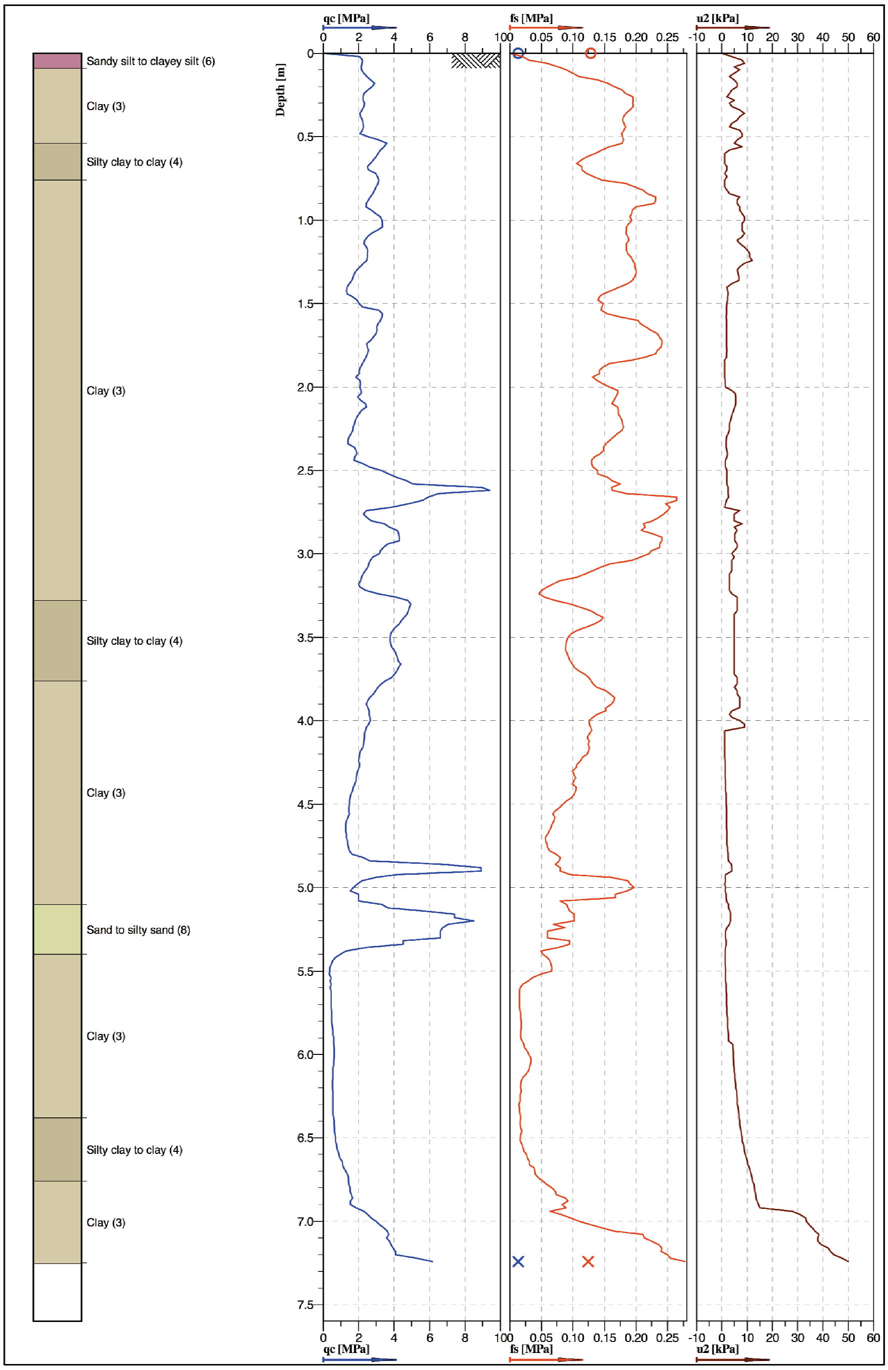

Fig. 1. The CPT protocol of hole No. 7 (Bagińska et. al., 2012)

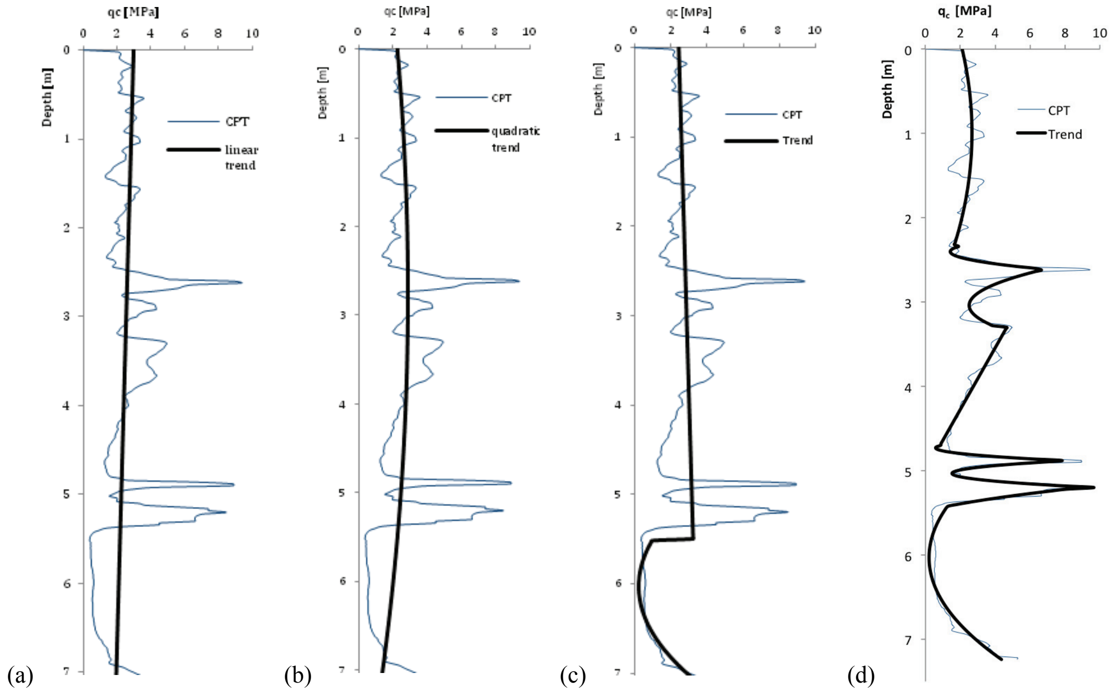

Fig. 2. Trend line: (a) linear trend for total CPT; (b) quadratic trend for total CPT; (c) linear trend for the part of the embankment and the quadratic trend for the part of the natural soil; (d) mix of quadratic and linear trends selected to best match to CPT

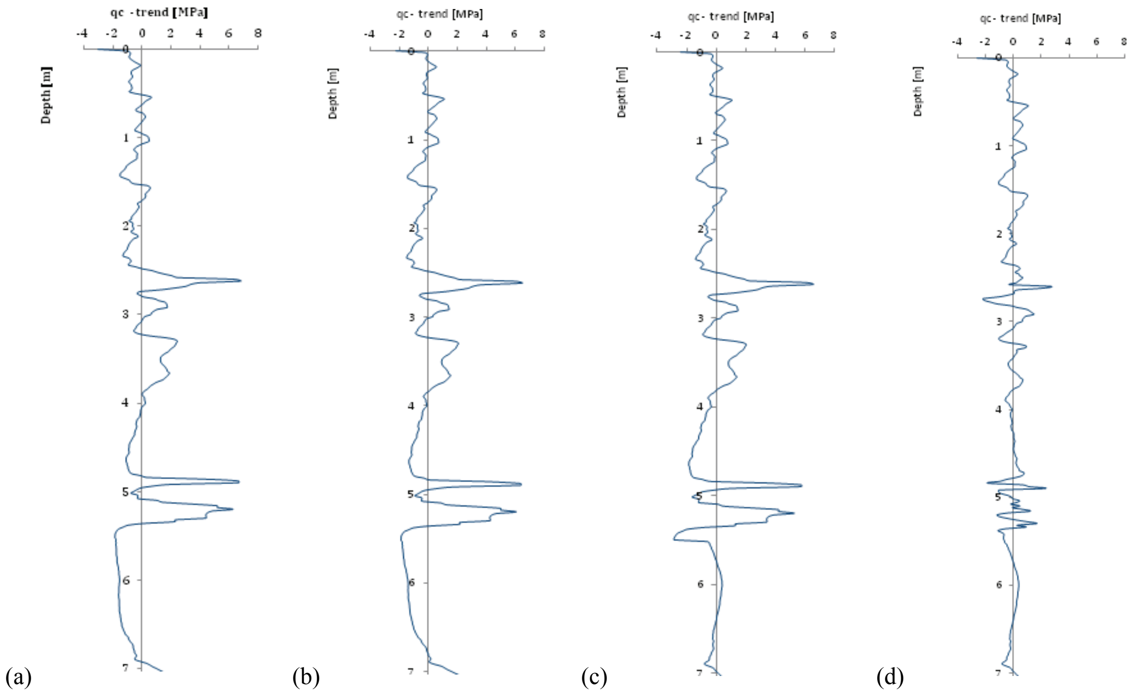

Fig. 3. The residual after de-trends: (a) linear for total CPT; (b) quadratic for total CPT; (c) linear for the part of embankment and quadratic for the part of the natural ground; (d) mix of quadratic and linear trends selected to best match to CPT

trends for embankment (0,0–5,5 m) and natural soil (5,5–7,2 m). The part of the natural soil was defined by the quadratic trend while the part of the embankment by the linear trend. The results are shown in Fig. 2c. In the plots, the thin lines indicate q_c values for CPT profiles, whereas the thicker lines indicate the average trend. To show the differences between results in Fig. 2 the trend mixed from linear and quadratic parts has been presented. The soil layers were divided by the trend best matching the CPT profiles.

The rest values $x(z)$ after de-trend cone tip resistance are shown in Figure 3. Inspection of Figure 3

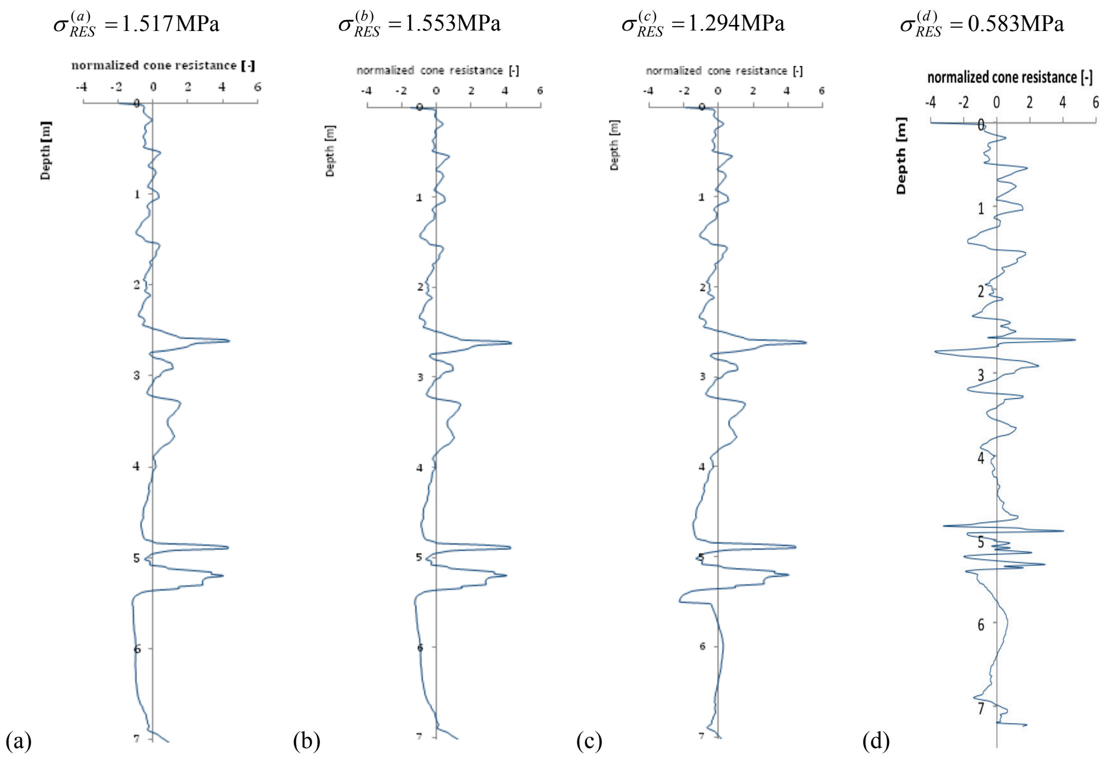

Fig. 4. The de-trend normalized cone resistance: (a) linear for total CPT; (b) quadratic for total CPT; (c) linear for the part of embankment and quadratic for the part of the natural ground; (d) mix of quadratic and linear trends selected to best match to CPT

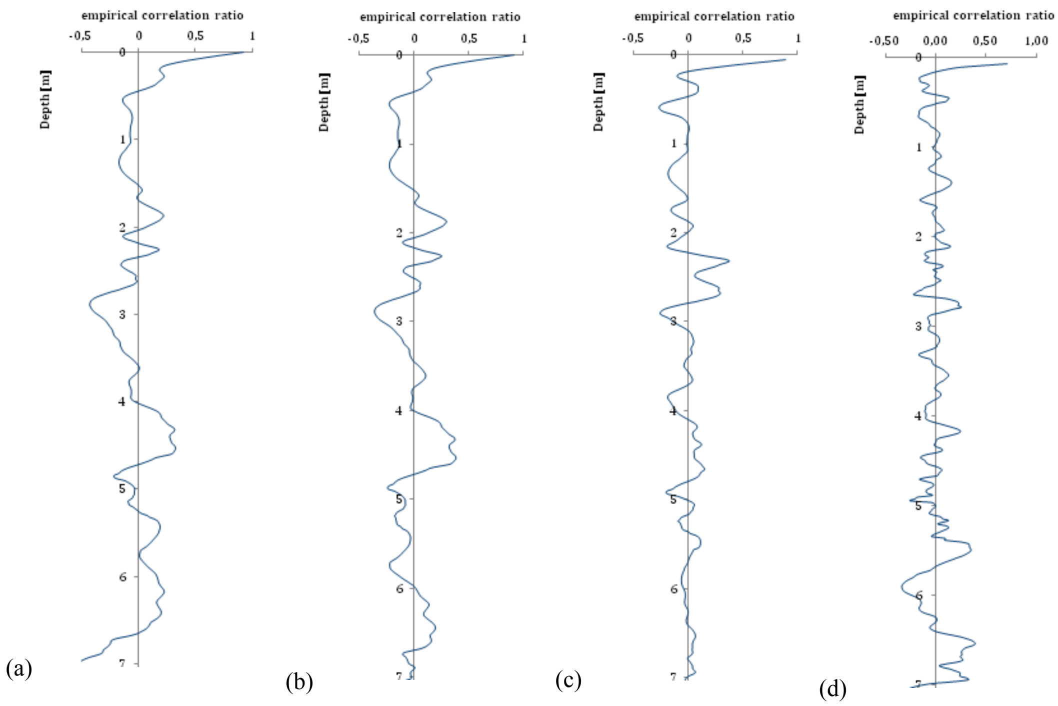

Fig. 5. The empirical correlation coefficient for total CPT after de-trends: (a) linear; (b) quadratic; (c) independent linear for the embankment and square for the natural soil; (d) mix of quadratic and linear trends selected to match to CPT

shows that the rest values $x(z)$ are similar especially in the part of the embankment.

For each analysis a rest standard distribution (σ_{RES}) was assumed and the rest values $x(z)$ were divided by it. The normalized de-trended CPT data is plotted in Fig. 4.

In all presented situations after de-trend CPT measurements the rest values statistics correspond to

Table 3. Mean value and standard deviation for the normalized residual

$\mu_{TN}^{(a)} = 8.91 \cdot 10^{-16}$	$\mu_{TN}^{(b)} = 2.38 \cdot 10^{-14}$	$\mu_{TN}^{(c)} = 1.91 \cdot 10^{-10}$	$\mu_{TN}^{(d)} = 1.09 \cdot 10^{-10}$
$\sigma_{TN}^{(a)} = 1$	$\sigma_{TN}^{(b)} = 1$	$\sigma_{TN}^{(c)} = 1$	$\sigma_{TN}^{(c)} = 1$

Table 4. The value of the vertical scale of fluctuation (Markov function)

	Vanmarcke Method	Rice Method
Linear de-trended CPT	$\theta_V^{(a)} = 0.28m$	$\theta_R^{(a)} = 0.23m$
Quadratic de-trended CPT	$\theta_V^{(b)} = 0.23m$	$\theta_R^{(b)} = 0.21m$
Linear de-trended CPT for the embankment and quadratic de-trended CPT for the natural soil	$\theta_V^{(c)} = 0.15m$	$\theta_R^{(c)} = 0.18m$
Mix of quadratic and linear trends selected to best match to CPT	$\theta_V^{(d)} = 0.08m$	$\theta_R^{(d)} = 0.14m$

Table 5. The value of the vertical scale of fluctuation (Gaussian function)

	Vanmarcke Method	Rice Method
Linear de-trended CPT	$\theta_V^{(a)} = 0.22m$	$\theta_R^{(a)} = 0.29m$
Quadratic de-trended CPT	$\theta_V^{(b)} = 0.20m$	$\theta_R^{(b)} = 0.26m$
Linear de-trended CPT for the embankment and quadratic de-trended CPT for the natural soil	$\theta_V^{(c)} = 0.15m$	$\theta_R^{(c)} = 0.23m$
Mix of quadratic and linear trends selected to best match to CPT	$\theta_V^{(d)} = 0.07m$	$\theta_R^{(d)} = 0.18m$

a standard random fields. The empirical correlation coefficients for each situation are shown in Fig. 5.

In Table 3 mean values (μ_{TN}) and standard deviations (σ_{TN}) of the normalized de-trend cone resistance are presented. The standard deviations equal zero while the mean values increase with the accuracy of matching the trend to CPT measurements.

Next tables (Table 4 and 5) are summarized analyses of the vertical scale of fluctuation, estimated using two most common models, the Markov correlation function in Table 4 and the Gaussian correlation function in Table 5. The scales of fluctuation have been estimated for different situations including different trend lines.

An important part of this investigation is to assess the two approaches used to estimate θ. The first approach is Vanmarcke method and the second one is Rice method. It can be noted that with increasing accuracy of matching the trend, the scales of fluctuations are decreasing. This trend seems to be correct. The second aspect of the observation is a similar scale of fluctuations for both applied methods of calculation. However, values presented in Table 4 and 5, do not allow clear value for further calculations. The assessment is considered by the comparison of the theoretical graph assuming the certain scale, estimated by the above method with the value of the correlation coefficient resulting from the research. The effects of such a comparison are shown in Figure 6. The first column describes theoretical coefficient of correlation by the Markov function whereas the second column by the Gaussian function.

Except for the case (d), differences between theoretical plots for Markov correlation model are relatively small. In case of the Gaussian correlation model the differences between methods are greater than in Markov model, but they similarly increase with the accuracy of matching the trends. Other assumed detail is the greater value of the scale of fluctuation estimated by the Rice method. An exception for that is the Markov correlation function for linear and quadratic trends where the Vanmarcke method gives higher values of the scale of fluctuation than the Rice method.

Figure 6 also shows that it is irrelevant to adapt the trend line. This is particularly evident in the case of the Gaussian correlation function. For linear and quadratic trend estimated for the total CPT the empirical correlation coefficient weakly matches the theoretical coefficient. However, in the case of the linear trend subtracted from the part of the embank-

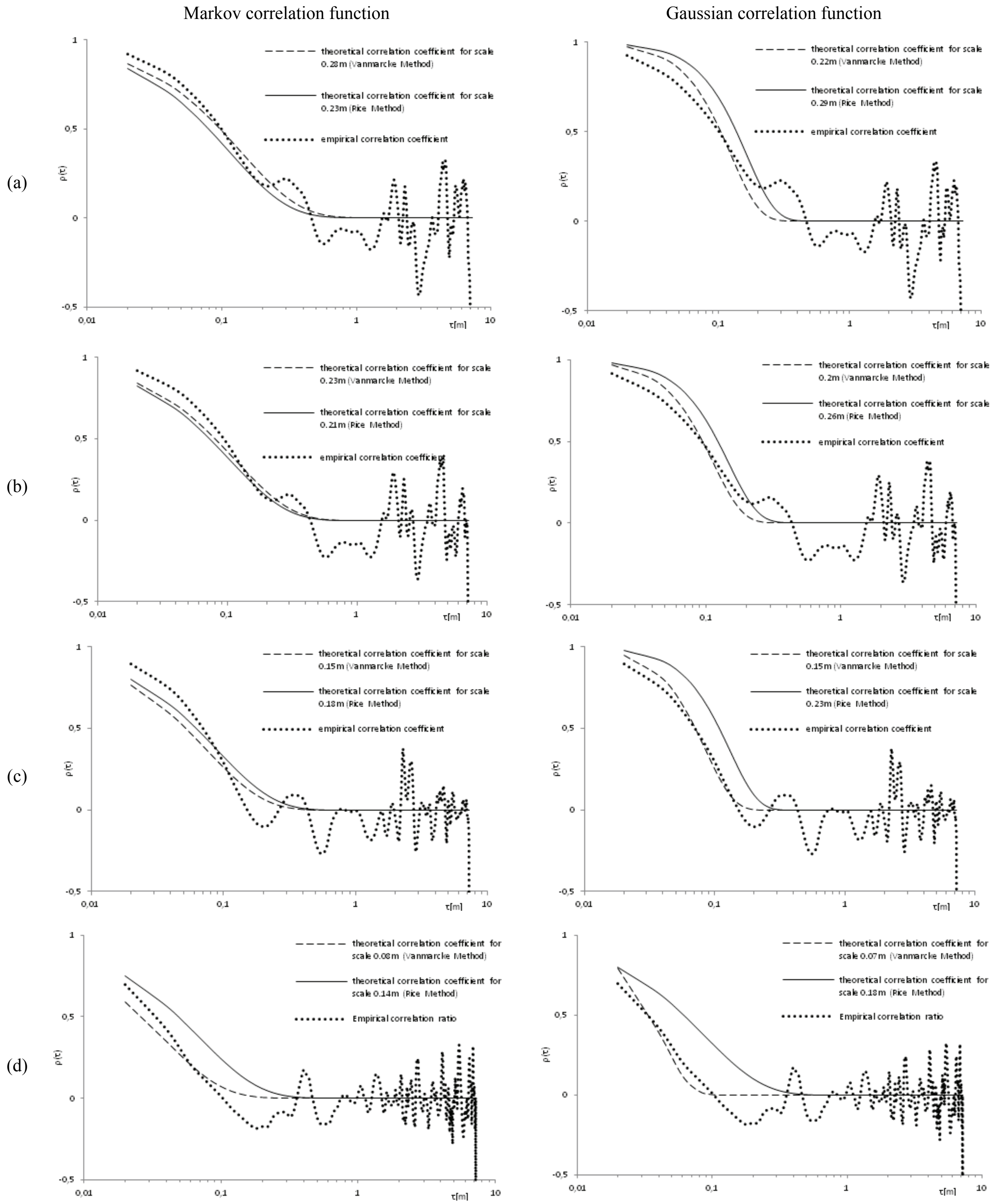

Fig. 6. Matching the empirical correlation ratio to the theoretical ratio after de-trend: (a) linear for total CPT; (b) quadratic for total CPT; (c) linear for the part of the embankment and the quadratic for the part of the natural soil; (d) mix of quadratic and linear trends selected to best match to CPT

ment and the square trend subtracted from part of the natural soil, the empirical Gaussian correlation coefficient adapts very well to the coefficient designated by Vanmarcke method. Similar situation can be observed in the case of mix trends according to the CPT measurements (case d).

3. CONCLUSION

To sum up, two methods for estimating scales of fluctuation have been presented and applied to a real case study. The goal of the paper is to show the differences between the methods.

The results of the study indicate that the value of the vertical scale of fluctuations decreases with increasing accuracy of the estimation of the trend. This effect is adequate for both methods. However, the results show how important it is to match the trend well to the CPT measurements. Four types of the trend have been tested in this paper. Two of them take into account the actual composition of the soil profile: the embankment (up to 5.5 m) and the natural soil under it and two of them do not. The scale of fluctuation value can be estimated for all situations, but for the linear (case a) and the quadratic trends (case b) adopted for total CPT the result are overstated.

The research of the correlation functions leads to conclusions that differences between the results are very small. The adoption of the correlation model does not affect the scale of fluctuations. However, it enables a better match between the empirical correlation coefficient and the theoretical one.

It can therefore be assumed that the Rice method gives greater value of the scale of fluctuation than the Vanmarcke method. Although the Vanmarcke method is more accurate, the Rice method requires less complicated numerical analysis leading to estimate the vertical correlation length. It leads to the conclusions that Rice method although rarely used in geotechnical engineering, might be as effective as the Vanmarcke method, if a large number of observations are available.

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