PRELIMINARY CALCULATION
OF IRREGULAR TRIGUY MAST SUPPORT

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Abstract: The instantaneous tangential rigidity (corresponding to the applied horizontal load) of a triguy support with unequal ropes is determined. The differences between the ropes can be due to the different location, both horizontal and vertical, of their anchoring foundations and to the different types of ropes used. The horizontal rigidity of such a support has no general axis of symmetry. Therefore a formula for this rigidity in any direction and a formula for extreme rigidity angles are given in this paper. A notion of the guy's eigenvalue, standing for the initial tension force above which the guy participates effectively in the support's rigidity, is introduced. A numerical example is provided.

Key words: *mast, irregular triguy support, initial tension values, forces in guys, numerical example*

1. INTRODUCTION

When designing masts, a situation can be encountered when one or several of the following circumstances occur (the letter symbols are illustrated in Fig. 1):

– unequal length s_i of guys at the same angle of chord inclination to level ϑ_i (due to the ground's inclination),
– unequal angle of inclination of chords relative to level ϑ_i (some of the guys have to be placed over the existing structures),
– unequal angles between chords in horizontal projection φ_i (there are structures to be bypassed),
– unequal axial rigidities of ropes E_iA_i (caused by repair conditions).

In such cases, an irregular triguy support, i.e., one which lacks initial horizontal axisymmetric rigidity and for which no directions of extreme horizontal rigidities can be determined in advance since they depend both on the initial tension values and the reaction values, is formed.

One should pay close attention to triguy supports since as the horizontal load on the mast increases in the conditions of insufficient initial tightening, their horizontal rigidity, particularly the lateral one (i.e., perpendicular to the wind load plane), decreases sharply as compared to the initial rigidity whereby the shaft may lose its stability.

It is always more difficult to calculate a mast with irregular multiguy supports than one with regular supports, since apart from the iteration of the shaft's static solution (not to mention the complexity of the formulas) there is the additional iteration of the support's solution aimed at finding the directions of extreme rigidities.
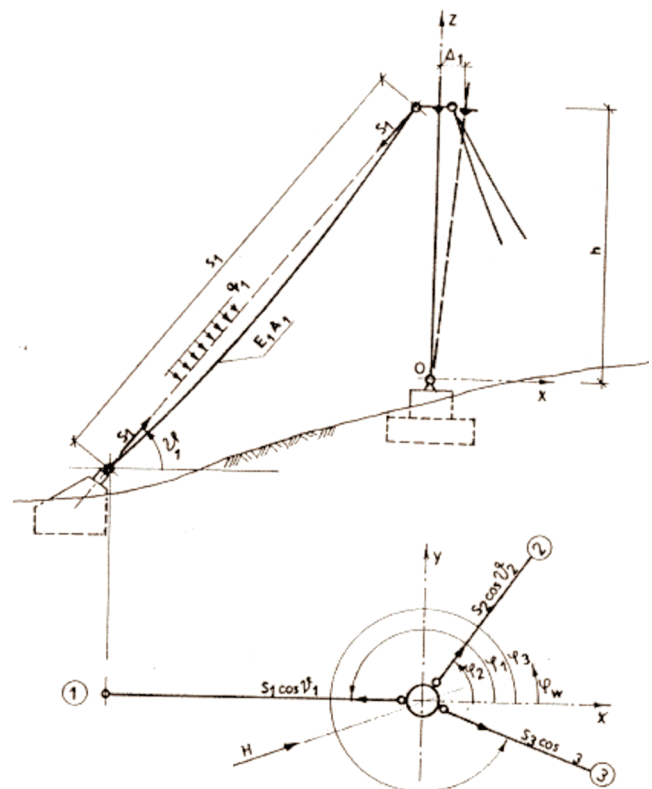

Fig. 1. A structural diagram of an irregular triguy support

The symbols and the formulas used in this paper for a single guy were adopted after [1], while the forces in the ropes wee calculated by Petersen's method [2].

2. INITIAL TENSION VALUES

Initial tension values S_{oi} ($i = 1, 2, 3$), generally different from one another, must fulfil static conditions of equilibrium in the horizontal plane, i.e.,

$$\sum_{i=1}^{3} S_{oi} \cos \vartheta_i \cos \varphi_i = 0; \quad \sum_{i=1}^{3} S_{oi} \cos \vartheta_i \sin \varphi_i = 0. \quad (1)$$

Up till now initial tension values for the ropes have been assumed according to construction recommendations [3], [6] and [7] which have a very general character. When they are assumed incorrectly, this prolongs the iterative design process. It is generally known that such initial tension values should be assumed for the adopted rope diameter that firstly, the greatest possible initial rigidity resulting in the shortening of the fundamental natural period, and secondly, sufficient operating rigidity in each direction are obtained. The guy's eigenvalue, standing for initial tension force S_{oi}^{*} from which the horizontal displacement of the guy's upper hitching point starts diminishing for a fixed value of the support's horizontal reaction, is adopted as the initial tension reference level. Starting with the conditions for the positive determination of coefficient K_i in the formula for the guy's horizontal displacement in the vertical plane, the sought eigenvalue for the upper hitching point will be obtained as the root of the following equation

$$B_i S_{oi}^{*3} - K_{Ti} S_{oi}^{*3} - A_{oi} = 0, \quad (2)$$

where

$$A_{oi} = g_{oi}^2 s_i^3 \cos \vartheta_i / 24, \quad (3a)$$

$$B_i = s_i / (E_i A_i \cos \vartheta_i), \quad (3b)$$

$$K_{Ti} = \varepsilon_{Tl}(T_l - T_o)s_i / \cos \vartheta_i - \varepsilon_{Tt} h(T_t - T_o) tg \upsilon_i, \quad (4)$$

g_{oi} – the rope's specific dead weight, kN/m,

T_l, T_t, T_o – the operating temperature of the rope and the shaft, respectively, and the assembly temperature;

ε_{Tl}, ε_{Tt} – coefficients of thermal expansions of the rope and the shaft, respectively.

For the range of climatic temperatures, equation (2) has only one real positive root. Its value can be determined most easily at assembly temperature since then

$$S_{oi}^{*3} = (A_{oi} / B_i)^{\frac{1}{3}}. \quad (5)$$

In design practice, one can use eigenvalue S^{*}_{oi}, calculated from formula (5) adding correction δS^{*}_{oi} for the rope's elongation due to the rise in temperature. On the basis of numerical calculations for guys made of type T.l*61 ropes [4], the following relation is proposed

$$S_{oi} = (A_{oi} / B_i)^{\frac{1}{3}} + 10^4 \varepsilon_{Tl}(T_l - T_o)(25 / d_i)^{0.42}(tg \vartheta_i)^{0.8}, \quad (6)$$

where d_i is the rope's catalog diameter in mm.

After three eigenvalues for three ropes have been calculated, the following is determined

$$S_{o,\min} > \max(S_{oi}^{*}). \quad (7)$$

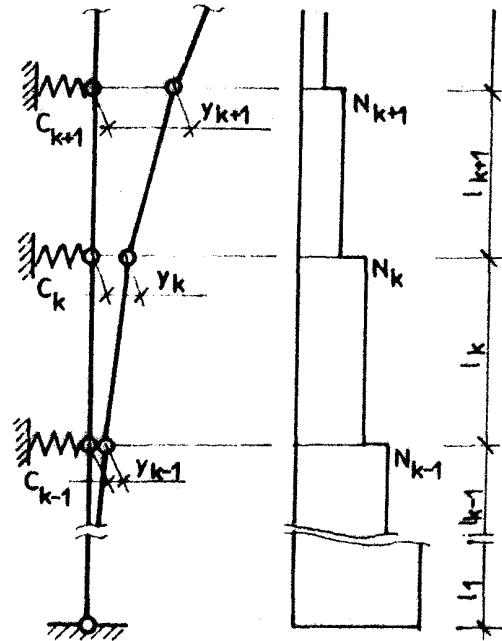The remaining two initial tension values must fulfil equations (1).

Fig. 2. The mast's shaft as a kinematic chain and a diagram of axial forces N_i

The support's minimum rigidity $C_{\min}$ in one of the extreme directions α_{extr}, which is to be calculated, is the second criterion for the adoption of proper initial tension values. This rigidity must ensure the stability, in a bifurcation sense, of the shaft modeled by a kinematic chain (the low estimate). Let us use

a linearized equation (the trigonometric functions of small arguments are replaced by the arguments themselves) of the k-th joint's equilibrium (Fig. 2)

$$N_k y_{k-1} / l_k - (N_k / l_k + N_{k+1} / l_{k+1} - C_{p,k}) y_k$$

$$+ N_{k+1} y_{k+1} / l_{k+1} = 0. \qquad (8)$$

Equations (8) written out for all the supports, after one parameter λ has been introduced for all axial forces N_k, form a system of equations homogenous in respect of joints displacement y_k. From this system's solvability condition (the zero value of the main determinant) one obtains an algebraic equation in respect of critical load parameter λ for the shaft with n elastic supports

$$\sum_{j=0}^{n} b_j \lambda^{n-j} = 0, \qquad (9)$$

where for a common case of a mast with four levels of guys (after omitting index min of C) we get

$$b_o = 1,$$

$$b_1 = -(C_1 + C_2 + C_3 + C_4)l_1 / N_1 - (C_2 + C_3 + C_4)l_2 / N_2 - (C_3 + C_4)l_3 / N_3 - C_4 l_4 / N_4,$$

$$b_2 = C_1(C_2 + C_3 + C_4)l_1 l_2 / N_1 N_2 + C_2(C_3 + C_4)l_2 l_3 / N_2 N_3 + C_3 C_4 l_3 l_4 / N_3 N_4 + (C_1 + C_2)(C_3 + C_4)l_1 l_3 / N_1 N_3 + (C_1 + C_2 + C_3)C_4 l_1 l_4 / N_1 N_4 + (C_2 + C_3)C_4 l_2 l_4 / N_2 N_4,$$

$$b_4 = -[C_1 C_2(C_3 + C_4)l_1 l_2 l_3 / N_1 N_2 N_3 + C_1 C_4(C_2 + C_3)l_1 l_2 l_4 / N_1 N_2 N_4 + C_3 C_4(C_1 + C_2)l_1 l_3 l_4 / N_1 N_3 N_4 + C_2 C_3 C_4 l_2 l_3 l_4 / N_2 N_3 N_4],$$

$$b_4 = C_1 C_2 C_3 C_4 l_1 l_2 l_3 l_4 / N_1 N_2 N_3 N_4$$

($C_4 = 0$ should be assumed for a mast with three levels of guys, whereas for a mast with two levels of guys one should adopt $C_3 = C_4 = 0$).

The support's initial rigidity in direction α will be

$$C_{o\alpha} = \sum_{i=1}^{3} C_{oi} \cos^2(\varphi_i - \alpha), \qquad (10)$$

where φ_i is the i-th guy's direction angle, whereas

$$C_{o\alpha} = S_{oi}^3 \cos \vartheta_i / (2A_{oi} + B_i S_{oi}^3). \qquad (11)$$

3. FORCES IN GUY

Using the conditions of the guy's equilibrium in horizontal plane (xy), i.e.,

$$H \cos \varphi_w + \sum_{i=1}^{3} S_i \cos \vartheta_i \cos \varphi_i = 0,$$

$$H \sin \varphi_w + \sum_{i=1}^{3} S_i \cos \vartheta_i \sin \varphi_i = 0, \qquad (12)$$

where H is the horizontal load on the joint, directed at angle φ_w to axis x which for reasons of convenience should be situated along guy No. 1, and the conditions of displacements inseparability (Fig. 3)

$$\sum_{i=1}^{3} \Delta_i = 0, \qquad (13)$$

where

$$\Delta_i = -A_i / S_i^2 + B_i S_i - K_i, \qquad (14)$$

the following algebraic septetic equation with unknown S_2 is obtained

$$\sum_{k=0}^{7} a_k S_2^{(7-k)} = 0, \qquad (15)$$

in which

$$a_7 = -A_2 S_{xy}^2 S_y^2,$$

$$a_6 = -2A_2 S_{xy} S_y F_1,$$

$$a_5 = -(A_1 S_y^2 + A_2 F_2 + A_3 S_{xy}^2) + (B_1 S_{xy} - B_3 S_y - K) S_{xy}^2 S_y^2,$$

$$a_4 = -2(A_1 S_y L + A_2 F_1 LM + A_3 S_{xy} M) + [B_1(2S_{xy}^2 L + 3S_{xy} S_y M) + B_2 S_{xy} S_y - B_3(2S_y^2 M + 3S_{xy} S_y L) - 2KF_1]S_{xy} S_y,$$

$$a_3 = -(A_1 L^2 + A_2 L^2 M^2 + A_3 M^2) + B_1 S_{xy} F_3 - 2B_2 S_{xy} S_y F_1 - B_3 S_y F_4 - KF_2,$$

$$a_2 = B_1 MF_4 + B_2 F_2 - B_3 LF_3 - 2KLMF_1,$$

$$a_1 = [B_1(3S_{xy} LM + 2S_y M^2) + 2B_2 F_1 - B_3(3S_y LM + 2S_{xy} L^2)]LM - KL^2 M^2,$$

$$a_0 = [B_1 M + B_2 - B_3 L)L^2 M^2,$$

$$F_1 = S_{xy}L + S_y M \,,$$

$$F_2 = S_{xy}^2 L^2 + S_y^2 M^2 + 4S_{xy}S_y LM \,,$$

$$F_3 = S_{xy}^2 L^2 + 3S_y^2 M^2 + 6S_{xy}S_y LM \,,$$

$$F_4 = 3S_{xy}^2 L^2 + S_y^2 M^2 + 6S_{xy}S_y LM \,,$$

$$K = K_1 + K_2 + K_3 \,, \quad [\text{m}],$$

$$A_i = q_i^2 s_i^3 /(24\cos\vartheta_i), \quad [\text{kN}^2/\text{m}] \tag{16}$$

$$K_i = -A_{oi}/S_{oi}^2 + B_i S_{oi} - K_{Ti}, \quad [\text{m}] \tag{17}$$

$$q_i = [(g_{oi}\cos\vartheta_i + q_{vi})^2 + q_{Hi}^2]^{\frac{1}{2}} \quad – \text{ the load perpen-}$$
dicular to the i-th guy's chord, kN/m;

$$q_{vi} = W_i \sin\vartheta_i \cos(\varphi_i + \varphi_w) \,,$$

$$q_{Hi} = W_i \sin(\varphi_i + \varphi_w) \,,$$

$$W_i = -q_{wi}[1 - \cos^2\vartheta_i \cos^2(\varphi_i + \varphi_w)]^{1/2} \,,$$

$$q_{wi} = \gamma_f q_k C_e C_x d_i \beta \quad – \text{ the wind load on a } d_i \text{ diameter}$$
guy at the height of $2h_i/3$, according to [6], kN/m;

$$L = \cos\vartheta_2 \sin\varphi_2 /(\cos\vartheta_3 \sin\varphi_3) \,, \tag{18}$$

$$M = (\cos\vartheta_2 \cos\varphi_2 - L\cos\vartheta_3 \cos\varphi_3)/\cos\vartheta_1 \,, \tag{19}$$

$$S_{xy} = (H\cos\varphi_w - S_y \cos\vartheta_3 \cos\varphi_3)/\cos\varphi_1 \,, \tag{20}$$

$$S_y = H\sin\varphi_w /(\cos\vartheta_3 \sin\varphi_3) \,, \tag{21}$$

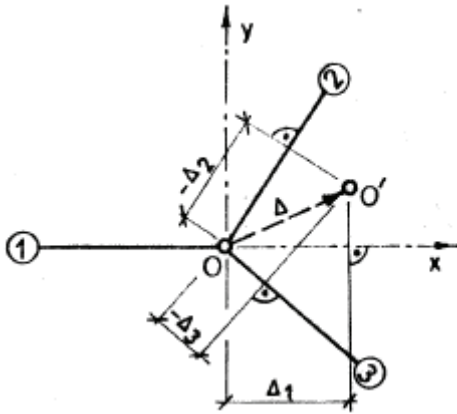q_k – the characteristic wind velocity pressure.

Fig. 3. The horizontal displacement
of the support by vector Δ_1

Equation (15) has at least one real root which follows from a fundamental theorem of algebra [5]. Numerous numerical calculations done for real systems show that there are three real roots of which only one is positive. Its value is in the interval

$$[1 + (|c|/a_7)^{1/k_1}, 1 + (|a_7|/a_0)^{1/k_2}],$$

where $|c|$ – the absolute value of the highest coefficient of negative polynomial $\left(\sum a_k S_2^{7-k}\right)$ and $\left(-\sum a_k S_2^k\right)$, respectively.

Taking into account force S_2 and expressions (16)–(19), the forces in the remaining ropes are calculated from the following conditions of equilibrium

$$S_1 = S_{xy} + S_2 M \,; \quad S_3 = -S_y - S_2 L \,. \tag{22}$$

The i-th guy's instantaneous rigidity C_i is calculated from formula (11) in which simply index "$_o$" is removed and the value calculated from formula (16) substituted for A_{oi}. The overall rigidity of the support in direction α is calculated in an analogous way, using formula (10).

The criterion condition for the extremum of function C_i yields

$$\alpha_{\text{extr}} = 0.5\left[\text{arctg}\left(\sum_{i=1}^{3} C_i \sin 2\varphi_i \Big/ \sum_{i=1}^{3} C_i \cos 2\varphi_i\right) + k\pi\right],$$

$$(k = 0, 1). \tag{23}$$

4. NUMERICAL EXAMPLE

An irregular triguy support at the temperature of assembly and under wind action in direction $\varphi_w = 0°$ is to be solved for the following geometry

$$h = 92 \text{ m},$$

$$\varphi_1 = 180° \,, \quad \varphi_2 = 72° \,, \quad \varphi_3 = 320° \,,$$

$$\vartheta_1 = 45° \,, \quad \vartheta_2 = 60° \,, \quad \vartheta_3 = 50° \,,$$

$$s_1 = 130 \text{ m} \,, \quad s_1 = 120 \text{ m} \,, \quad s_1 = 140 \text{ m} \,.$$

All the guys are made of type T1*61 rope of $d = 25$ mm diameter, for which $A = 375$ mm^2, $g_o = 0.0315$ kN/m and $E = 145$ GPa.

Using (18) and (19), we calculate:

$$L = -1.150, \quad M = 1.019.$$

The guys' characteristics calculated from formulas (3) and their eigenvalues calculated from formula (5) are as follows

$$A_{01} = 62.658; \quad A_{02} = 36.864; \quad A_{03} = 71.148;$$

$$B_1 = 3.381 \cdot 10^{-3}; \quad B_2 = 4.414 \cdot 10^{-3}; \quad B_3 = 4.004 \cdot 10^{-3};$$

$$S_{01}^* = 26.463 \text{ kN}; \quad S_{02}^* = 20.289 \text{ kN}; \quad S_{03}^* = 26.095 \text{ kN}.$$

Table 1. Initial tension values $S_{\alpha i}$ of the guys and their initial rigidities C_{oi}

No.	S_{01}	S_{02}	S_{03}	C_{01}	C_{02}	C_{03}	K_1	K_2	K_3
	kN			kN/m			m		
1	13.2	12.95	14.90	12.22	13.04	13.66	−0.315	−0.162	−0.261
2	26.46	25.97	29.86	69.71	57.95	68.80	0	0.060	0.040
3	50	49.05	56.40	161.31	99.23	134.04	0.144	0.201	0.203
4	100	98.10	112.80	201.67	111.31	157.71	0.332	0.429	0.446

From formula (7) we obtain: $S_{0,\min} = S_{01} = 26.463$ kN.

The calculations were done for all four values of initial tightening $S_{01} = 13.2$, 26.463, 50 and 100 kN, (the first value was assumed for comparison with the obtained rigidity of the support, measured by the horizontal displacement at given reaction). Forces S_{02} and S_{03} were calculated from formulas (1), initial rigidities C_{oi} of the ropes along their horizontal directions from formulas (11), and coefficients K_i from formula (17) into which $K_{Ti} = 0$ should be put since a design situation at the temperature of assembly is considered. The calculated values have been compiled in table 1.

The directions and the extreme initial rigidities calculated using formulas (23) and (10), respectively, have been gathered in Table 2.

Table 2. Extreme rigidities $C_{\max}$ and $C_{\min}$
of the support considered and the corresponding directions

S_{01}	α_{extr}^{max}	α_{extr}^{min}	$C_{o\alpha}^{max}$	$C_{o\alpha}^{min}$
kN	grades		kN/m	
13.2	−27° 32′45″	62° 27′15″	22.994	15.934
26.4	−22° 02′53″	67° 57′07″	122.451	76.039
50	−17° 37′01″	72° 22′59″	261.140	133.439
100	−16° 19′01‴	73° 43′59″	317.269	152.413

It was assumed that the support considered would carry horizontal force $H = 20$, 40, and 80 kN, depending on the span length of the shaft's spans adjacent to the support, the type of the shaft's structure and the installations mounted on the shaft.

Figure 4 illustrates the distribution of the support's horizontal rigidity C_α for all the initial tension values S_{01} and loads H, while Fig. 5 shows the relationship between force $S1$ and displacement Δ_1, depending on load H for fixed initial tension values S_{01} (Fig. 5a) and on initial tension S_{01} for fixed forces H (Fig. 5b).

To make the diagrams in Fig. 4a more legible, the distributions of initial rigidity $C_{o\alpha}$ have been plotted in the figure's upper part, whereas those of current rigidity C_α under initial tightening $S_{01} = 13.2$ and 100 kN (for the initial tightening of 100 kN all curves for $H = 20$, 40 and 80 kN practically coincide) have been grouped in its lower part. Figure 4b again shows diagrams $C_{o\alpha}$ in its top part and the distributions of current rigidity C_α under initial tightenings $S_{01} = 26.4$ and 50 kN in its bottom part.

Changes in the maximum and minimum rigidities in relation to the initial rigidities depending on joint load H are illustrated in Fig. 6.
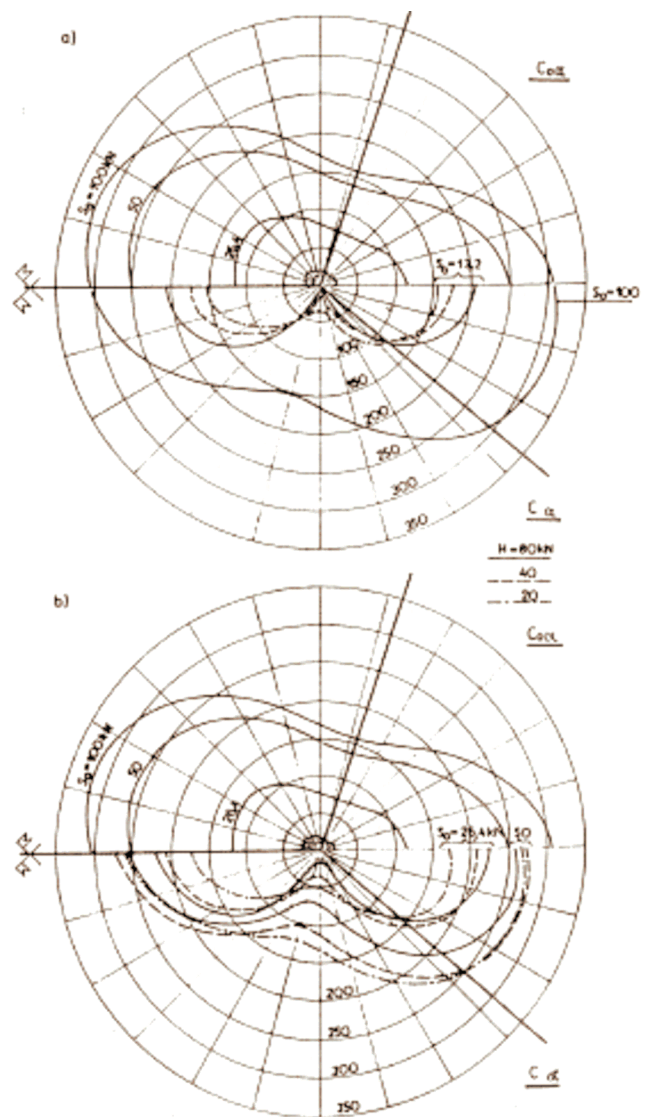

Fig. 4. The distribution of horizontal initial $C_{o\alpha}$
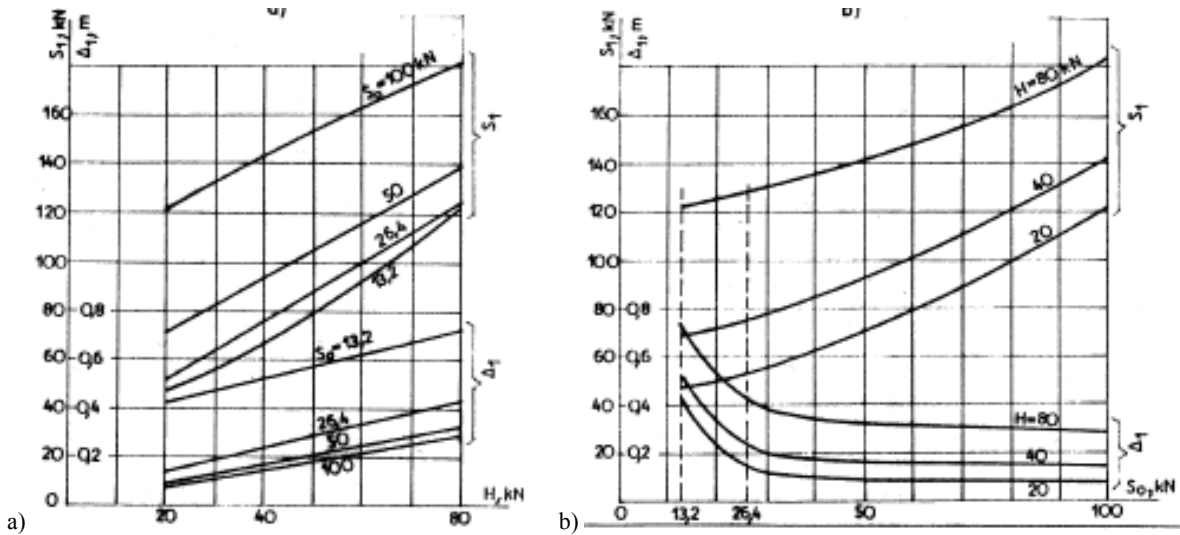and current C_α rigidity (description in the text)

Fig. 5. The relationship between force S_1 in the first rope, the shaft's horizontal displacement Δ_1 in this rope's plane and:
(a) horizontal load H on the joint under different initial tension values S_0,
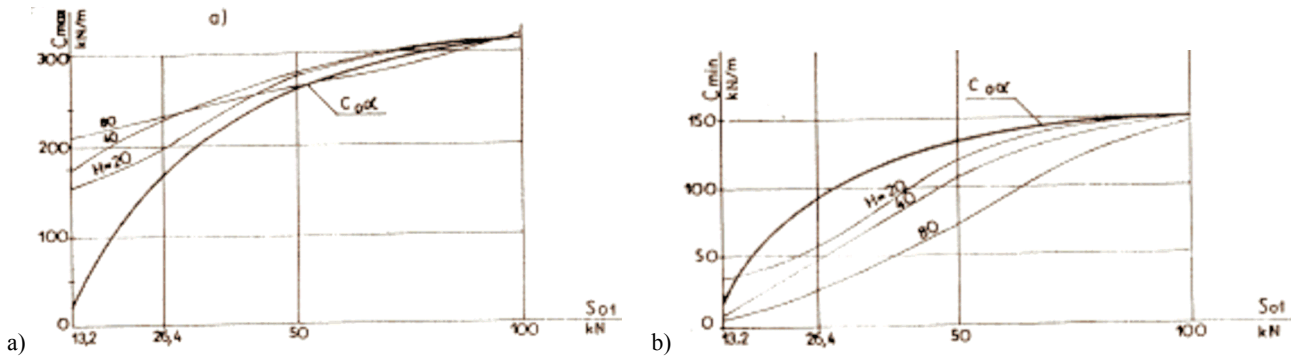(b) initial tension S_0 of the rope under different horizontal loads H on the joint

Fig. 6. Changes in the extreme rigidities for the support considered

5. CONCLUSIONS

The formulas derived in this paper allow one to calculate the characteristics of any triguy support. The force of initial tightenings is the basic parameter of the support's rigidity. The initial tension of the least tightened rope in the support should not be lower than the guy's eigenvalue calculated from formula (5). Otherwise, such a support is very sensitive to horizontal load H. This manifests itself mainly in a wide variation of the support's rigidity in the plane of loading, which increases the number‾ of iterations in the shaft's static solution, and in a sharp decrease in the rigidity in the lateral plane whereby the shaft may lose its stability.

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