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Complex analysis of uniaxial compressive tests of the Mórágy granitic rock formation (Hungary)

Abstract: Understanding the quality of intact rock is one of the most important parts of any engineering projects in the field of rock mechanics. The expression of correlations between the engineering properties of intact rock has always been the scope of experimental research, driven by the need to depict the actual behaviour of rock and to calculate most accurately the design parameters. To determine the behaviour of intact rock, the value of important mechanical parameters such as Young's modulus (E), Poisson's ratio (v) and the strength of rock (σ_{cd}) was calculated. Recently, for modelling the behaviour of intact rock, the crack initiation stress (σ_{ci}) is another important parameter, together with the strain (ε). The ratio of Young's modulus and the strength of rock is the modulus ratio (M_R), which can be used for calculations. These parameters are extensively used in rock engineering when the deformation of different structural elements of underground storage, caverns, tunnels or mining opening must be computed. The objective of this paper is to investigate the relationship between these parameters for Hungarian granitic rock samples. To achieve this goal, the modulus ratio ($M_R = E/\sigma_c$) of 50 granitic rocks collected from Bátaapáti radioactive waste repository was examined. Fifty high-precision uniaxial compressive tests were conducted on strong (σ_c >100 MPa) rock samples, exhibiting the wide range of elastic modulus (E = 57.425–88.937 GPa), uniaxial compressive strength (σ_c = 133.34–213.04 MPa) and Poisson's ratio (v = 0.18–0.32). The observed value (M_R = 326–597) and mean value of M_R = 439.4 are compared with the results of similar previous researches. Moreover, the statistical analysis for all studied rocks was performed and the relationship between M_R and other mechanical parameters such as maximum axial strain ($\varepsilon_{a,\ max}$) for studied rocks was discussed.

1 Introduction

Rock engineering properties are considered to be the most important parameters in the design of groundworks. Two important mechanical parameters, uniaxial compressive strength (σ_c) and elastic modulus of rock (E), should be estimated correctly. There are different empirical relationships between σ_c and E obtained for limestones, agglomerates, dolomites, chalks, sandstones and basalts [1, 2, 3], among the others.

Hypothetical stress–strain curves for three different rocks are presented in Fig. 1 by Ramamurthy et al. [4]. Based on the figure, curves OA, OB and OC represent three stress–strain curves with failure occurring at A, B and C, respectively. According to their sample, curves OA and OB have the same modulus but different strengths and strains at failure, whereas the curves OA and OC have the same strength but different modulus and strains at failure. It means, neither strength nor modulus alone could be chosen to represent the overall quality of rock. Therefore, strength and modulus together will give a realistic understanding of the rock's response to engineering usage. This approach of defining the quality of intact rocks was proposed by Deere and Miller [5] considering the modulus ratio (M_R), which is defined as the ratio of tangent modulus of intact rock (E) at 50% of failure strength and its compressive strength (σ_c).

The modulus ratio $M_R = E/\sigma_c$ between the modulus of elasticity (E) and uniaxial compressive strength (σ_c) for intact rock samples varies from 106 to 1,600 [6]. For most rocks, M_R is between 250 and 500 with average M_R = 400, $E = 400\ \sigma_c$.

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Palchik [7] examined the M_R values for 11 heterogeneous carbonate rocks from different regions of Israel. The investigated dolomites, limestones and chalks had weak to very strong strength with a wide range of elastic modulus. He found that M_R is closely related to the maximum axial strain ($\varepsilon_{a,\,max}$) at the uniaxial strength of the rock (σ_c) and the following relationship was found (Fig. 2):

$$M_R = \frac{2k}{\varepsilon_{a,\,max}\left(1 + e^{-\varepsilon_{a,\,max}}\right)} \quad (1)$$

where k is the conversion coefficient equal to 100 and $\varepsilon_{a,max}$ is in %. When M_R is known, $\varepsilon_{a,\,max}$ (%) is obtained from Eq. (1) as

$$\varepsilon_{a,\,max} = \frac{k}{M_R - 0.46k} \quad (2)$$

The expansion of the expression $2/(1 + e^{\varepsilon_{a,\,max}})$ using Taylor's theorem shows the value of $2/(1 + e^{\varepsilon_{a,\,max}}) = 1 + 0.46\,\varepsilon_{a,\,max}$ [8].

The goal of this paper is to check Eq. (1) for Hungarian granitic rocks as well as to study the relationships between characteristic compressive stress level, strain and mechanical properties. These granitic rock samples were investigated previously by Vásárhelyi et al. [8] using multiple failure state triaxial tests.

Figure 1: Hypothetical stress–strain curves [4].

Figure 2: Relationship between modulus ratio (M_R) and maximum axial strain ($\varepsilon_{a,\,max}$) using different carbonate rocks [7].

2 Laboratory investigations and analyses

Laboratory samples originated from research boreholes deepened in carboniferous Mórágy granite formation during the research and construction phases of deep geological repository of low- and intermediate-level radioactive waste. This granite formation is a carboniferous intruded and displaced Variscan granite pluton situated in South-West Hungary. The main rock types are mainly microcline megacryst-bearing, medium-grained, biotite monzogranites and quartz monzonites [9] (see Fig. 3). In spatial viewpoint, the monzogranitic rocks contain generally oval shaped, variably elongated monzonite enclaves (predominantly amphibole–biotite monzonites, diorites and syenites) of various sizes (from a few centimetre to several 100 metres) reflecting the mixing and mingling of two magmas with different composition. Feldspar quartz-rich leucocratic dykes
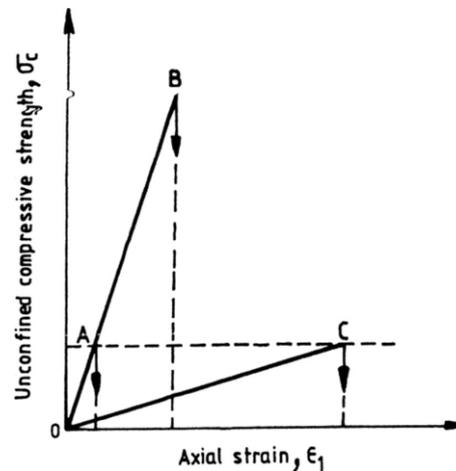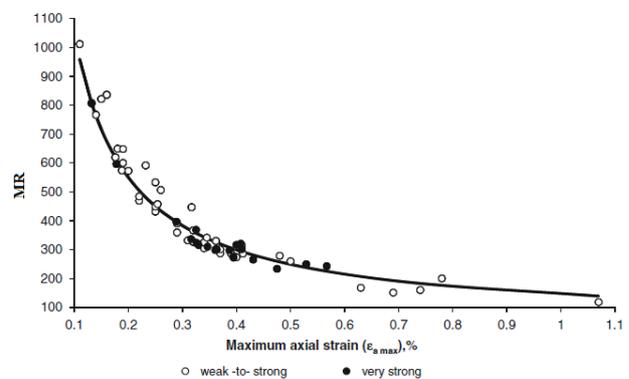
belonging to the late-stage magmatic evolution and Late Cretaceous trachyte and tephrite dykes cross cut all of the previously described rock types [10]. In general, fractured but fresh rock is common which is sparsely intersected by fault zones with few metre thick clay gauges. Intense clay mineralisation in the fault cores indicates a low-grade hydrothermal alteration.

The samples were tested by using a computer-controlled servo-hydraulic machine in continuous load control mode. The magnitude of loading was settled in kilonewton with 0.01 accuracy, and the rate of loading was 0.6 kN/s. Axial and tangential deformation was measured by strain gauges, which measures the deformation between 1/4 and 3/4 of the sample's height.

Fifty uniaxial compressive tests were performed in the rock mechanics laboratory at RockStudy Ltd. The NX ($d = 50$ mm)-sized cylindrical rock samples having the ratio of $L/d = 2/1$ (here L and d are the length and diameter of a sample, respectively) were prepared (see
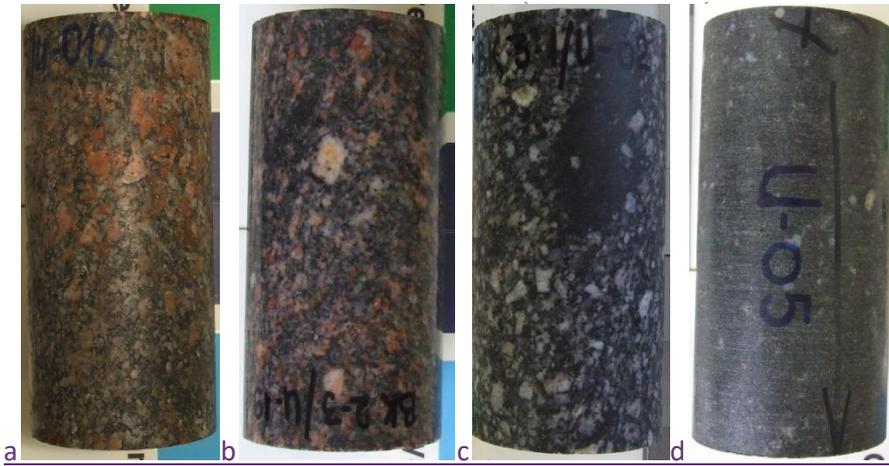
Figure 3: Main types of rock samples. (a, b) Megacryst-bearing, medium-grained, biotite monzogranites. (c) Medium-grained, biotite monzogranites with elongated monzonitic enclaves. (d) Quartz monzonite.

Figure 4: A prepared sample in the beginning of the UCS test.

Fig. 4). Mechanical properties of granitic rock samples are summarised in Table 1.

UCS, uniaxial compressive strength

Table 1 summarizes the value of elastic modulus (*E*), crack damage stress (σ_{cd}), uniaxial compressive strength (σ_c), Poisson's ratio (*v*), crack initiation stress (σ_{ci}), axial failure strain ($\varepsilon_{a,\ max}$), maximum volumetric strain (ε_{cd}), crack initiation strain (ε_{ci}) and M_R for each of the studied 50 samples.

The values of elastic modulus (*E*) and Poisson's ratio (*v*) were calculated by using linear regressions along linear portions of stress–axial strain curves and radial strain–axial strain curves, respectively. The values of

crack initiation stress (σ_{ci}) and crack damage stress (σ_{cd}) were calculated based on the following methods:

2.1 Onset dilatancy method

In this method, [11], crack initiation threshold is visible on the axial–volumetric strain curve (Fig. 5) when it diverges from the straight line. In practice, small deviation of the stress–volumetric strain curve from the straight line can make some difficulties to define one point determining the threshold of crack initiation.

2.2 − Crack volumetric strain method

Martin and Chandler [12] proposed that crack initiation could be determined using a plot of crack volumetric strain versus axial strain (Fig. 6). Crack volumetric strain ε_{Vcr} is calculated as a difference between the elastic volumetric strain ε_{Vel} and volumetric strain ε_V determined in the test,

$$\varepsilon_V = 2\varepsilon_l + \varepsilon_a \tag{3}$$

$$\varepsilon_{Vcr} = \varepsilon_V - \varepsilon_{Vel} \tag{4}$$

$$\varepsilon_{Vel} = \frac{1-2v}{E}\left(\sigma_1 + 2\sigma_3\right) \tag{5}$$

ε_a and ε_l are the axial and lateral strain; σ_1 and σ_3 are the axial and confining stress and *E* and *v* are the Young's modulus and Poisson's ratio, respectively.

Table 1: *Mechanical properties of investigated Mórágy granitic rock samples.*

Rock sample	υ	E	ε_{ci}	σ_{ci}	ε_{cd}	σ_{cd}	$\varepsilon_{a,\,max}$	σ_c	M_R
	(-)	(GPa)	(%)	(MPa)	(%)	(MPa)	(%)	(MPa)	(-)
BeR-6_U-10	0.24	74.776	0.030	50.73	0.091	152.244	0.278	181.05	413.0
BeR-7_U-02	0.21	71.612	0.037	50.37	0.095	145.15	0.34	174.80	409.7
BeR-7_U-04	0.25	74.447	0.037	59.61	0.063	131.70	0.33	183.39	405.9
BeR-8_U-01	0.22	63.357	0.060	59.84	0.120	165.89	0.29	184.48	343.4
BeR-10_U-08	0.21	66.129	0.025	30.06	0.044	77.30	0.22	137.14	482.2
BeR-10_U-18	0.23	72.794	0.048	64.89	0.078	148.24	0.2	148.39	490.6
BeR-10_U-20	0.23	63.787	0.035	39.28	0.087	133.75	0.27	156.74	407.0
BeR-11_U-08	0.23	68.950	0.054	80.82	0.104	168.94	0.31	204.23	337.6
BeR-12_U-02	0.22	79.660	0.029	34.36	0.08	128.84	0.18	133.34	597.4
BK1-1_U-12	0.23	70.153	0.036	51.74	0.076	131.50	0.23	172.74	406.1
BK1-3_U-01	0.32	72.891	0.037	79.97	0.053	121.05	0.28	184.59	394.9
BK1-3_U-03	0.19	69.164	0.065	71.75	0.14	132.66	0.22	133.62	517.6
BK1-3_U-04	0.18	71.860	0.045	47.93	0.113	112.28	0.18	153.60	467.8
BK1-3_U-08	0.23	70.137	0.059	80.36	0.147	142.79	0.22	172.55	406.5
BK1-3_U-12	0.25	57.425	0.066	67.99	0.13	134.11	0.27	135.14	424.9
BK2-1_U-03	0.21	74.228	0.057	74.61	0.09	131.78	0.19	146.65	506.2
BK2-3_U-07	0.28	77.332	0.036	59.84	0.068	119.12	0.19	143.71	538.1
BK2-3_U-15	0.22	80.365	0.035	48.57	0.090	160.74	0.24	178.41	450.5
BK2-3_U-18	0.2	73.819	0.069	80.22	0.11	153.84	0.23	159.16	463.8
BK2-4_U-02	0.2	76.820	0.06	88.12	0.106	177.32	0.26	205.62	373.6
BK2-4_U-04	0.21	77.709	0.045	60.07	0.090	130.57	0.20	155.49	499.8
BK2-5_U-02	0.25	77.866	0.038	62.63	0.070	134.14	0.23	166.29	468.3
Bkf-1_U-03	0.24	77.665	0.050	50.46	0.070	120.14	0.30	161.63	480.5
Bkf-2_U-03	0.22	60.602	0.065	76.84	0.118	164.66	0.39	180.93	334.9
Bkf-4_U-03	0.22	79.856	0.042	60.29	0.083	142.38	0.24	179.28	445.4
Bkf-5_U-02	0.24	79.818	0.034	53.90	0.067	135.29	0.20	169.67	470.4
Bl-112_U-02	0.21	72.897	0.029	37.88	0.093	144.19	0.20	164.59	442.9
Bp-4_U-05	0.25	76.992	0.041	69.46	0.1	181.85	0.24	187.69	410.2
Bp-4B_U-01	0.21	69.800	0.042	49.25	0.109	159.85	0.36	184.45	378.4
Bp-4B_U-05	0.23	76.237	0.033	49.37	0.076	148.77	0.28	170.10	448.2
Bp-4B_U-13	0.27	77.924	0.049	74.11	0.096	170.28	0.25	177.91	438.0
Bp-4B_U-17	0.24	74.648	0.045	60.27	0.083	162.61	0.26	181.43	411.4
Bp-4B_U-19	0.22	77.182	0.058	80.43	0.100	160.13	0.25	190.48	405.2
Bp-4B_U-23	0.24	74.683	0.053	80.00	0.077	137.96	0.24	165.23	452.0
Bp-5_U-19	0.25	73.506	0.031	49.73	0.056	121.48	0.23	149.76	490.8
Bp-5_U-21	0.25	80.159	0.040	70.45	0.064	137,00	0.26	171.46	467.5
Bx-81_U-03	0.22	65.782	0.045	53.44	0.088	130.84	0.29	149.28	440.7
Bx-82_U-01	0.25	82.940	0.046	80.51	0.085	162.08	0.27	180.33	459.9
Bx-82_U-03	0.29	84.949	0.024	49.99	0.044	120.612	0.2	166.87	509.1
Bx-83_U-01	0.26	72.864	0.030	60.56	0.067	150.321	0.26	169.70	429.4
Bx-83_U-03	0.25	78.072	0.057	90.36	0.095	182.085	0.37	212.42	367.5
Bx-84_U-01	0.25	80.669	0.047	79.90	0.073	147.6	0.23	178.07	453.0

Continued **Table 1:** *Mechanical properties of investigated Mórágy granitic rock samples.*

Rock sample	υ	E	ε_{ci}	σ_{ci}	ε_{cd}	σ_{cd}	$\varepsilon_{a,\,max}$	σ_c	M_R
	(-)	(GPa)	(%)	(MPa)	(%)	(MPa)	(%)	(MPa)	(-)
Bx-84_U-03	0.27	81.144	0.039	69.38	0.062	138.183	0.26	166.94	486.1
Bx-101_U-02	0.24	76.994	0.042	71.53	0.058	112.5	0.19	142.49	540.3
Bx-101_U-04	0.26	79.300	0.048	60.58	0.091	160.96	0.23	163.19	485.9
Bz-921_U-01	0.21	71.574	0.056	68.79	0.121	164.573	0.3	192.80	371.2
Bz-942_U-01	0.23	73.511	0.053	73.43	0.11	182.66	0.28	198.58	370.2
Bz-1221_U-01	0.2	69.540	0.049	58.25	0.100	165.836	0.29	213.04	326.4
Bz-1311_U-01	0.3	88.937	0.035	75.93	0.060	163.371	0.23	206.48	430.7
Bz-1351_U-01	0.25	67.053	0.034	50.86	0.080	145.566	0.28	159.97	419.2
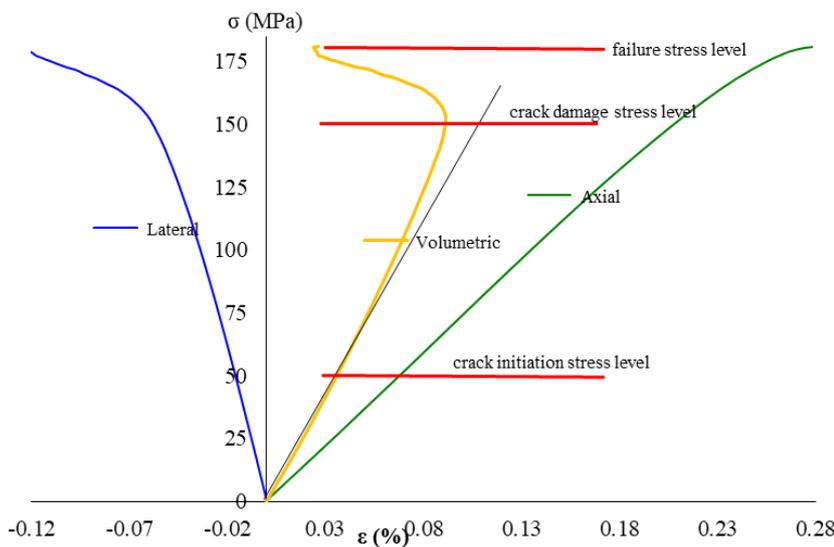
Figure 5: Axial stress–volumetric strain curve with the threshold of crack initiation and crack damage and failure stress for Hungarian granitic sample (uniaxial compression case).
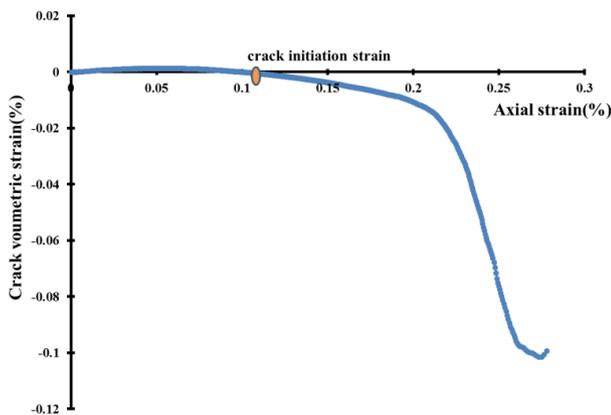
Figure 6: Crack volumetric strain method for crack initiation threshold determination for Hungarian granitic rock sample (uniaxial compression case).

Crack volumetric strain is calculated on the basis of these two elastic constants and is strongly sensitive to its value. This is probably why this method does not give objective values.

2.3 Change of Poisson's ratio method

Diederichs [13] proposed a method of crack initiation threshold identification based on the change of Poisson's ratio. The onset of crack initiation can be identified by the analysis of the relationship of Poisson's ratio, evaluated locally, to the log of the axial stress (Fig. 7).

However, in this paper, the results obtained from the first method were used for further analysis. The reason is
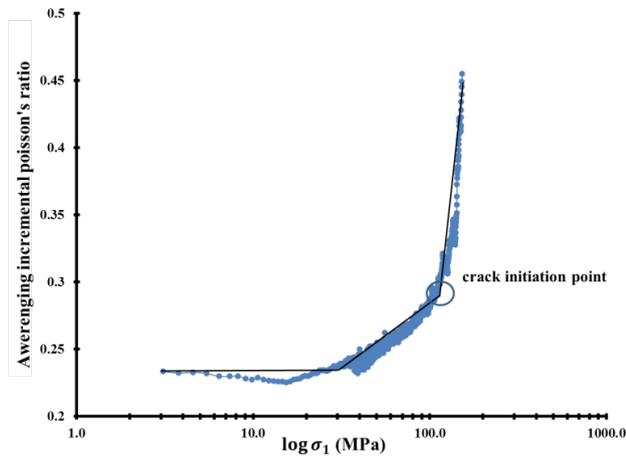
Figure 7: Poisson's ratio method for crack initiation threshold determination for Hungarian granitic rock sample (uniaxial compression case).
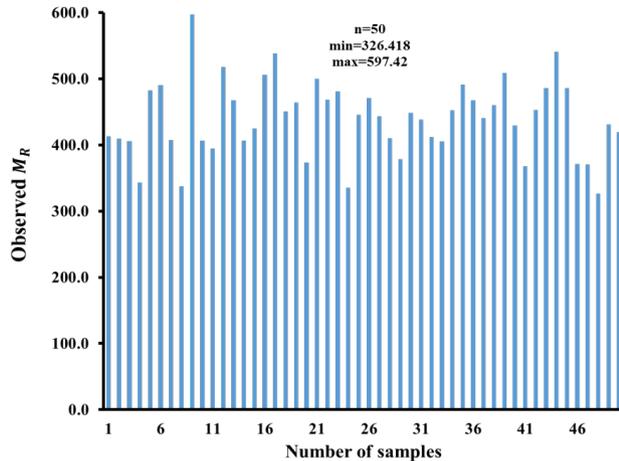

Figure 8: Observed values of modulus ratio (M_R) in each of 50 examined rock samples.

that, based on the findings by Cieslik [14], this method gives more precise results for granitic rock samples.

Table 1 also summarizes that the value of M_R in each of 50 studied granitic rock samples is between 326.4 and 597.4 with the mean of 439.4. The range of M_R obtained by Deere [15] is between 250 and 700 with the mean of 420 for limestone and dolomites. The range of M_R obtained by Palchik [7] is between 60.9 and 1011.4 with the mean value of 380.5 for carbonated rock samples. The mean value of M_R in this study is similar to the mean value of M_R obtained by Deere [15] and Palchik [7]. Fig. 8 shows the value of M_R for all studied samples in this study. As shown in Fig. 5, the range of M_R =326.4–597.4, observed in this study, is less than the range of M_R obtained by Deere [15] and Palchik [7].

The ranges of the elastic modulus (E), Poisson's ratio (v), crack damage stress (σ_{cd}) and uniaxial compressive strength (σ_c), axial failure strain ($\varepsilon_{a,\,max}$) and maximum volumetric strain (ε_{cd}), crack initiation stress (σ_{ci}) and crack initiation strain (ε_{ci}) for the studied 50 samples are presented as follows:

57.425 GPa $< E <$ 88.937 GPa
$0.18 < v < 0.32$
30 MPa $< \sigma_{ci} <$ 90 MPa
77 MPa $< \sigma_{cd} <$ 182 MPa
133.34 MPa $< \sigma_c <$ 213.04 MPa
$0.02 < \varepsilon_{ci} < 0.06$
$0.18 < \varepsilon_{a,\,max} < 0.19$
$0.04 < \varepsilon_{cd} < 0.14$

The ranges of $\frac{\varepsilon_{a,\,max}}{\varepsilon_{cd}}$ and $\frac{\sigma_{cd}}{\sigma_c}$ and $\frac{\sigma_{ci}}{\sigma_c}$ ratios are 1.49–5.28 and 0.5–0.9 and 0.2–0.5, respectively. These values are different from the values of $\frac{\sigma_{cd}}{\sigma_c}$ = 0.5–1.0 and $\frac{\varepsilon_{a,\,max}}{\varepsilon_{cd}}$ =1.51–6.91 obtained by Palchik [7]. They are also different from the values of $\frac{\sigma_{cd}}{\sigma_c}$ = 0.71–0.84 obtained by Brace et al. [11], Bieniawski [16], Martin [17], Pettitt et al. [18], Eberhardt et al. [19], Heo et al. [20] and Katz and Reches [21] for granites, sandstones and quartzites. The range of $\frac{\sigma_{ci}}{\sigma_c}$ for most rocks falls in the range of 0.3–0.5.

3 Effect of mechanical properties on MR value

3.1 Relationship between M_R, σ_c and E for all granitic rock samples

The relationship between uniaxial compressive strength (σ_c), M_R and E is shown in Fig. 9. It illustrates that how uniaxial compressive strength influences M_R and E for all studied rock samples.

As it is clear, the elastic modulus is related to σ_c, with R^2 = 0.06 very small. It also demonstrates that increase in the value of σ_c from 133 to 213 MPa does not influence E value. It can be seen from Fig. 9 that M_R is related to σ_c, with R^2 = 0.61. These values, however, are different from the values found by Palchik [7] for carbonated rocks. In his studies, the elastic modulus is partly related to uniaxial compressive strength with R^2 = 0.55 and increase in the value of σ_c does not influence M_R value (R^2= 0.021 is very small).
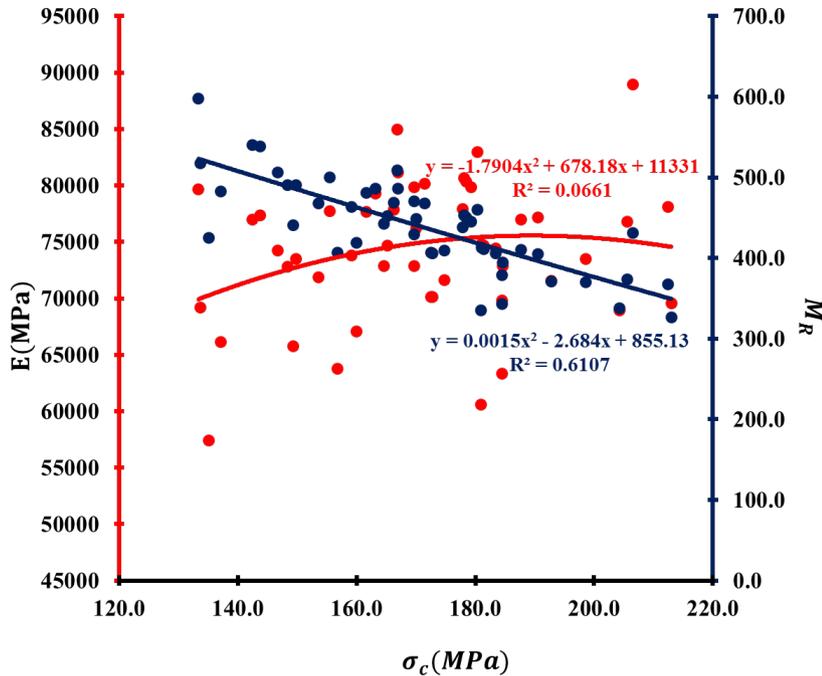
Figure 9: Influence of uniaxial compressive strength (σ_c) on elastic modulus (E) and the value of M_R for all studied samples.

3.2 Relationship between M_R value and different strain and stress of the rock

The calculated values are compared with the international published relationships.

3.2.1 Relationship between M_R and maximum axial strain ($e_{a, max}$) for all studied samples

The observed and analytical (Eq. 1) relationships between $\varepsilon_{a, max}$ and M_R for all rock samples exhibiting $\varepsilon_{a, max} < 1\%$ are plotted in Fig. 10. It is clear that the calculated Diederichs Eq. (1) and observed values of M_R for studied rock samples are similar. Fig. 11 presents the relative and root-mean-square errors between the calculated Diederichs Eq. (1) and observed M_R at $\varepsilon_{a, max} < 1\%$. As it is clear, the relative error (ς, %) for studied samples is between 0.28% and 25% and root-mean-square error is ($\chi = 50$). Comparing the values with the results obtained by Palchik [7] for carbonated rock samples, the relative error is between 0.08% and 10.8% and the root-mean-square error is 43.6.

The relative $\left(\varsigma, \%\right)$ and root-mean-square (χ) errors between the observed and calculated parameter Π have been calculated as:

$$\varsigma_{(m)} = \frac{2 \left| \Pi_{\mathrm{obs}(j)} - \Pi_{\mathrm{cal}(j)} \right|}{\Pi_{\mathrm{obs}(j)} + \Pi_{\mathrm{cal}(j)}} \times 100 \qquad (6)$$

$$\chi_{(m)} = \sqrt{\frac{\sum\limits_{j=1}^{n} \left[\Pi_{\mathrm{obs}(j)} - \Pi_{\mathrm{cal}(j)} \right]^2}{n - 1}} \qquad (7)$$

where $\Pi_{\mathrm{obs}(j)}$ is the observed value of parameter in the jth sample, here is M_R, $\Pi_{\mathrm{cal}(j)}$ is the calculated value of parameter in the jth sample, $j = 1, 2,...,n$, is the number of tested samples, here is 50.

3.2.2 Relationship between M_R and maximum volumetric strain e_{cd} for all studied samples

The observed values between M_R and ε_{cd} are plotted in Fig. 12. As it is clear, these parameters are partially related ($R^2 = 0.2$) for studied rock samples. Palchik [7] however, found a good relationship ($R^2 = 0.85$) between these two parameters for carbonated rock samples.
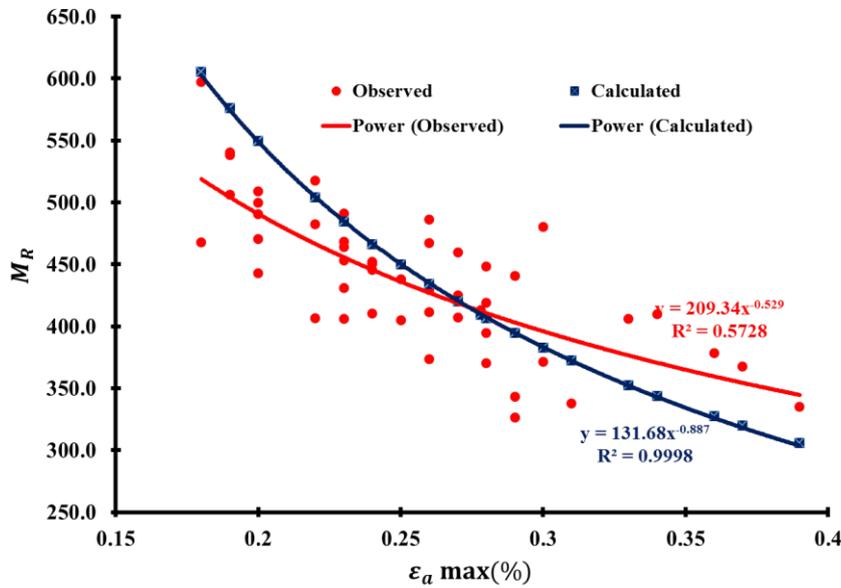
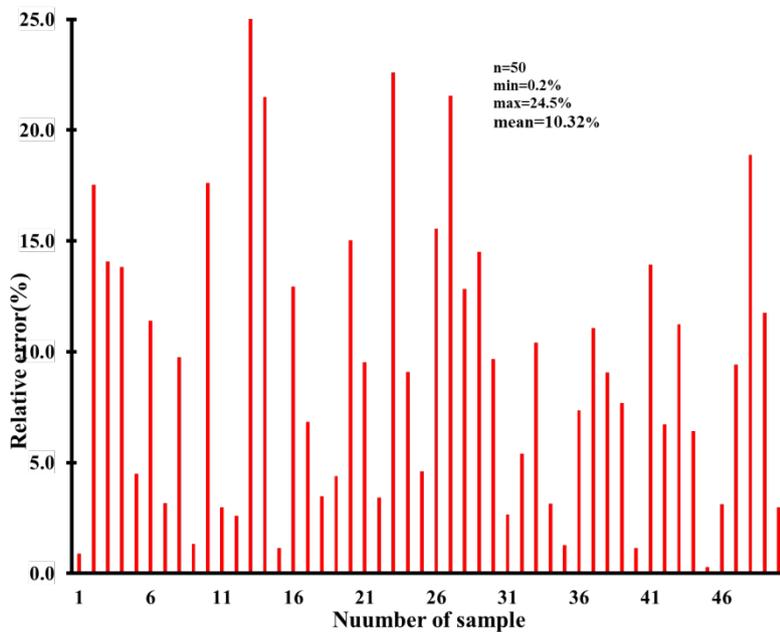**Figure 10:** Observed and analytical (Eq. 1) relationship between $\varepsilon_{a,\,max}$ and M_R.

Figure 11: Relative $(\varsigma, \%)$ and root-mean-square (χ) errors between calculated (Eq. 1) and observed M_R.

3.2.3 Relationship between M_R and crack damage stress s_{cd} for all studied samples

Fig. 13 shows the relationship between M_R and crack damage stress (σ_{cd}) for all studied rock samples. As it can be seen, there is a relationship ($R^2 = 0.41$) between these two parameters.

3.2.4 Relationship between M_R and $\frac{\sigma_{cd}}{\sigma_c}$ for all studied samples

The relationship between M_R and $\frac{\sigma_{cd}}{\sigma_c}$ is presented in Fig. 14. As it can be seen, there is practically no relationship ($R^2 = 0.0009$) between them.

3.2.5 Relationship between M_R and $\frac{\varepsilon_{a,max}}{\varepsilon_{cd}}$ for all studied samples

As shown in Fig. 15, there is practically no correlation between these two values.

3.2.6 Relationship between $\frac{\sigma_{cd}}{\sigma_c}$ and $\frac{\varepsilon_{a,max}}{\varepsilon_{cd}}$ for all studied samples

The relationship between $\frac{\sigma_{cd}}{\sigma_c}$ and $\frac{\varepsilon_{a,max}}{\varepsilon_{cd}}$ is presented in Fig. 16. As it can be seen, there is a relationship ($R^2 = 0.4$). Palchik [22], however, found the relationship ($R^2 = 0.7$) for carbonated rock samples.

3.2.7 Relationship between M_R and crack initiation stress (σ_{ci})

The relationship between M_R and crack initiation stress (σ_{ci}) is presented in Fig. 17. As it can be seen, there is practically no relationship between them ($R^2 = 0.08$).

3.2.8 Relationship between M_R and crack initiation strain (ε_{ci})

Fig. 18 shows the relationship between M_R and crack initiation strain (ε_{ci}). As it is shown, there is a relationship ($R^2 = 0.13$).

3.2.9 Relationship between M_R and $\frac{\sigma_{ci}}{\sigma_{cd}}$

Fig. 19 presents the relationship between M_R and $\frac{\sigma_{ci}}{\sigma_{cd}}$. As it is clear, there is practically no relationship ($R^2 = 0.03$).

3.2.10 Relationship between M_R and $\frac{\varepsilon_{ci}}{\varepsilon_{cd}}$

Fig. 20 shows the relationship between M_R and $\frac{\varepsilon_{ci}}{\varepsilon_{cd}}$. As it can be seen, there is practically no relationship ($R^2 = 0.06$).

3.2.11 Relationship between $\frac{\sigma_{ci}}{\sigma_{cd}}$ and $\frac{\varepsilon_{cd}}{\varepsilon_{ci}}$

Fig. 21 shows the relationship between $\frac{\sigma_{ci}}{\sigma_{cd}}$ and $\frac{\varepsilon_{cd}}{\varepsilon_{ci}}$. As it can be seen, there is a relationship ($R^2 = 0.54$).
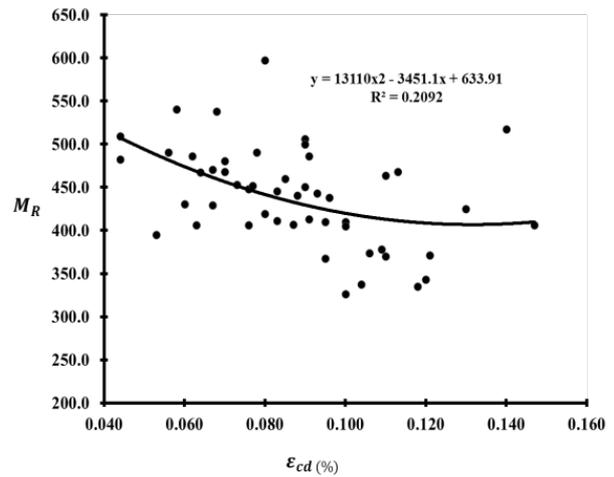

$$y = 13110x2 - 3451.1x + 633.91$$
$$R^2 = 0.2092$$

Figure 12: Relationship between M_R and ε_{cd} (%).

$$y = -0.014x2 + 2.2457x + 414.17$$
$$R^2 = 0.4192$$
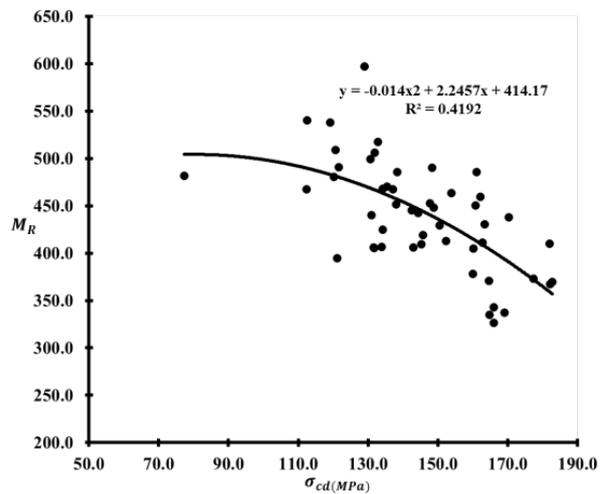
Figure 13: Relationship between M_R and σ_{cd}.

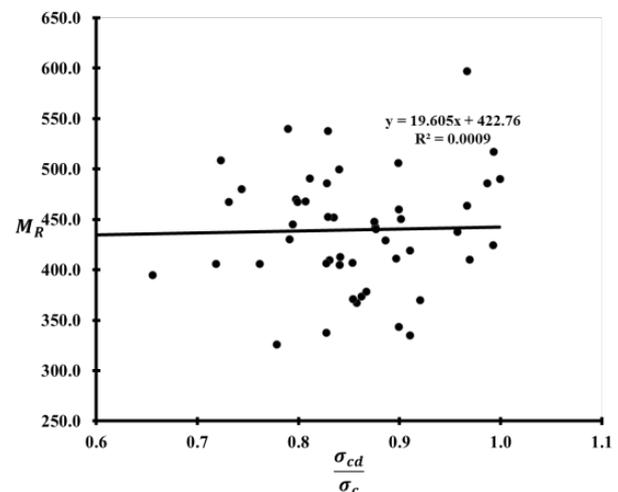

$$y = 19.605x + 422.76$$
$$R^2 = 0.0009$$

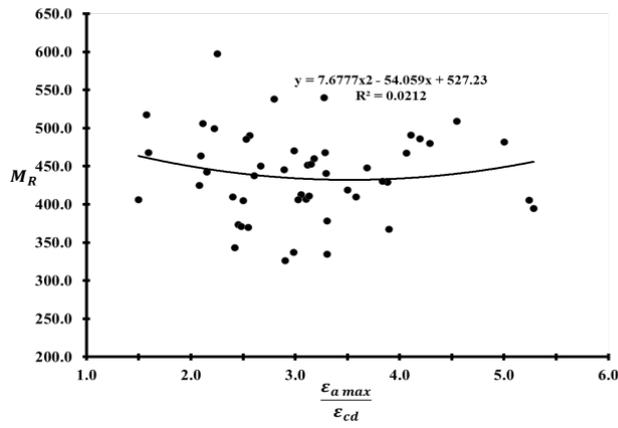**Figure 14:** Relationship between M_R and $\frac{\sigma_{cd}}{\sigma_c}$.

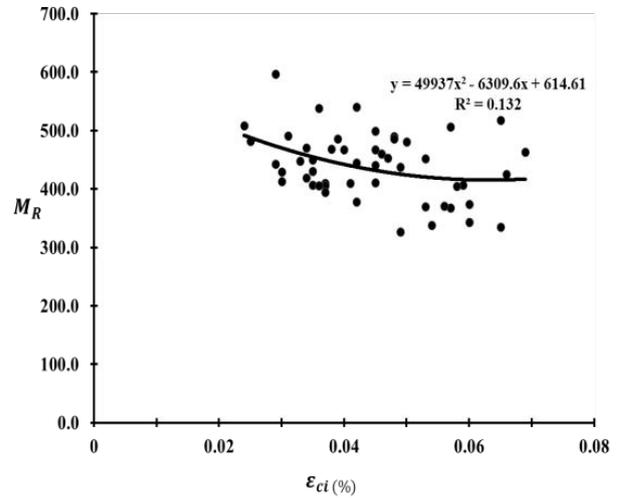**Figure 15:** Relationship between M_R and $\dfrac{\varepsilon_{a,max}}{\varepsilon_{cd}}$.
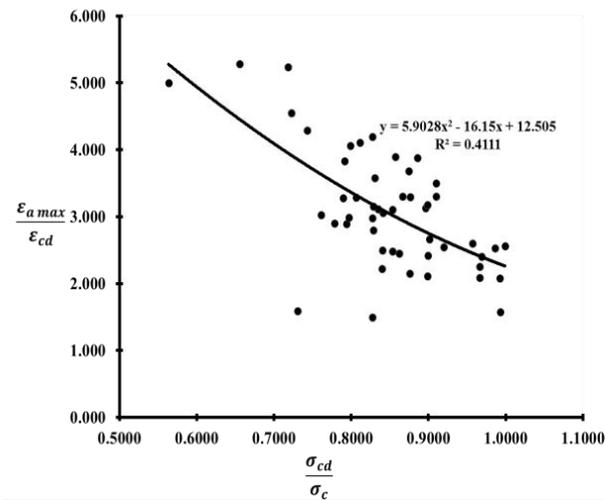

Figure 16: Relationship between $\dfrac{\sigma_{cd}}{\sigma_c}$ and $\dfrac{\varepsilon_{a,max}}{\varepsilon_{cd}}$.

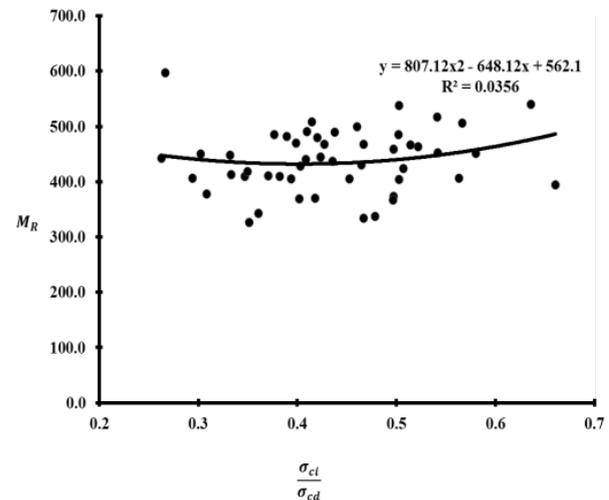**Figure 17:** Relationship between M_R and crack initiation stress (σ_{ci}).
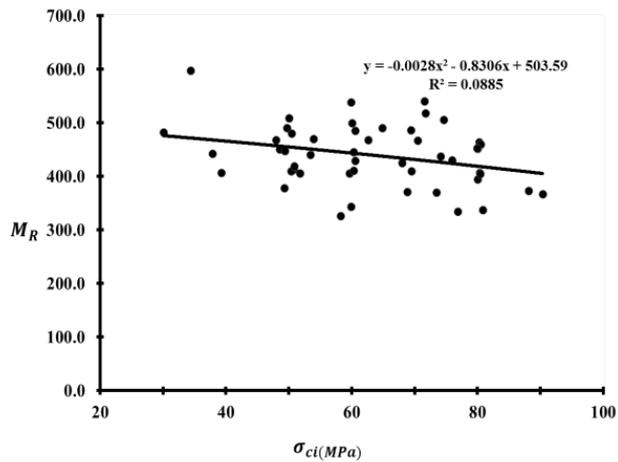

Figure 18: Relationship between M_R and crack initiation strain (ε_{ci}).

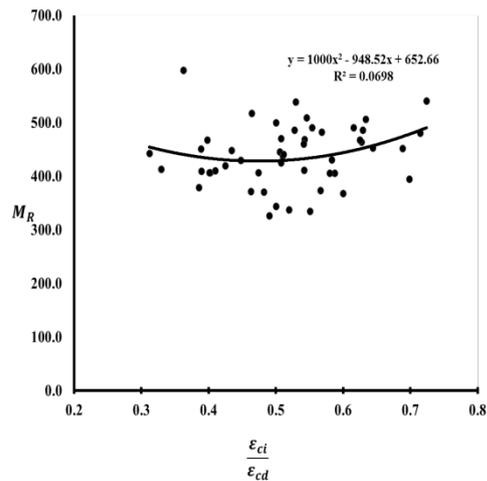**Figure 19:** Relationship between M_R and $\dfrac{\sigma_{ci}}{\sigma_{cd}}$.

Figure 20: Relationship between M_R and $\dfrac{\varepsilon_{ci}}{\varepsilon_{cd}}$.
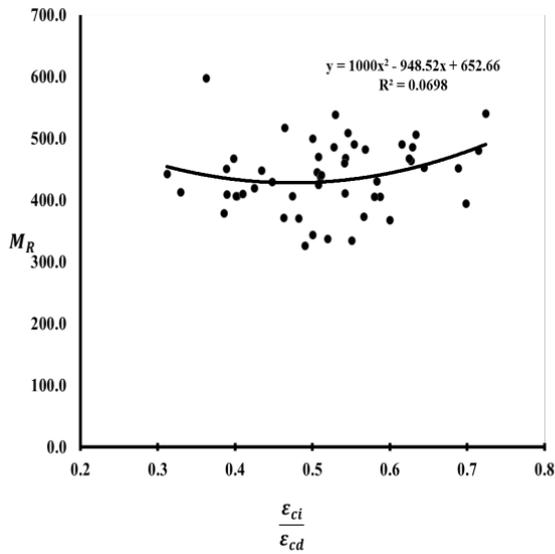
Figure 21: Relationship between $\frac{\sigma_{ci}}{\sigma_{cd}}$ and $\frac{\varepsilon_{cd}}{\varepsilon_{ci}}$.

4 Results and discussions

The laboratory compressive tests, statistical analysis and empirical and analytical relationships have been used to estimate the values of $M_R = E/\sigma_c$ and its relationship with other mechanical parameters for granitic rocks. Studied rock samples exhibited the wide range of mechanical properties (57.425 GPa < E < 88.937 GPa, 0.18 < v < 0.32, 77.3 MPa < σ_{cd} < 212.42 MPa, 133.34 MPa < σ_c < 213.04 MPa, 0.18 < $\varepsilon_{a\,max}$ < 0.19, 0.04 < ε_{cd} < 0.14). From the results of this study, the following main conclusions are made:

- The mean value of $M_{R\,mean}$ = 439 for all granitic rock samples observed in this study and the mean value of $M_{R\,mean}$ = 420 obtained by Deere [15] for limestone and dolomite and the mean value of $M_{R\,mean}$ = 380.5 obtained by Palchik [7] for carbonated rock samples are similar. However, the range of M_R = 326.42–597.42 obtained in this study is narrower than the range of M_R = 250–700 obtained by Deere [15] and the range of M_R = 60–1,600 obtained by Palchik [7].
- The observation confirms that there is no general empirical correlation (with reliable R^2) between elastic modulus (E) and uniaxial compressive strength (σ_c), M_R and maximum volumetric strain (ε_{cd}), M_R and crack damage stress σ_{cd}.
- The analytical l relationship (Eq. 1) between $\varepsilon_{a\,max}$ and M_R offered by Palchik [7] or carbonated rock samples was investigated for granitic rock samples in this study. It is observed that this relationship can also be used for granitic rocks. The relative error (ς, %) for studied samples is between 0.2% and 24.5% and root-

mean-square error is ($\chi = 50$). Comparing the values with the result obtained by Palchik [7] for carbonated rock samples, the relative error is between 0.08% and 10.8% and the root-mean-square error is 43.6.

- The observed correlation between M_R and ε_{cd} for studied granitic rock sample is $R^2 = 0.2$. Palchik [7], however, found a good relationship ($R^2 = 0.85$) between these two parameters for carbonated rock samples.
- It is established that there is a correlation between $\frac{\sigma_{ci}}{\sigma_{cd}}$ and $\frac{\varepsilon_{cd}}{\varepsilon_{ci}}$ with $R^2 = 0.54$.
- Based on the obtained results, there is practically no relationship between M_R and $\left(\frac{\varepsilon_{a,\,max}}{\varepsilon_{cd}}\right)$, $\left(\frac{\sigma_{cd}}{\sigma_c}\right)$, (σ_{ci}), $\left(\frac{\sigma_{ci}}{\sigma_{cd}}\right)$, $\frac{\varepsilon_{cd}}{\varepsilon_{ci}}$; however, there is a relationship between M_R and $(\varepsilon_{cd}),(\sigma_{cd})$ and (ε_{ci}).

Notably, for a more precise and fundamental description of the mechanical behaviour of rock, one should apply non-equilibrium continuum thermodynamics along the lines of Asszonyi et al. [23, 25] and beyond. These relationships can be used for determining the mechanical parameters of the rock mass, as well [24, 26].

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