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Optimisation of absorber parameters in the case of stochastic vibrations in a bridge with a deck platform for servicing pipelines

Abstract: The paper focuses on the problem of optimising the cooperation between a dynamic vibration absorber (DVA) and a structure. The authors analyse a road beam bridge equipped with a working platform (deck) used to service pipelines installed on the structure. The paper studies the problem of choosing the optimal parameters for damping absorbers that reduce the random vibration of a beam subjected to a random sequence of moving forces with a constant velocity. The stochastic properties of the load are modelled by means of a filtering Poisson process. A single-degree-of-freedom (SDOF) absorber model with a multi-degree-of-freedom (MDOF) primary structure model are is considered.

Keywords: urban network infrastructure; dynamic vibration absorber (DVA); vibration effect; dynamic analysis; Poisson process; random vibration.

1 Introduction

The problem of reducing the level of vibrations in various constructions has been considered for many years. This is especially the case for classic bridge structures when an additional platform on which installations, for example, pipes with hazardous gas, are installed.

Numerous ways and means of preventing unacceptable vibrations are known, with one of these methods being use of various vibration absorbers. Their application has a special role in civil engineering due

to the fact that they can be used during a construction's design, as well as later on to modify the structure. Using absorbers is an interesting option for reducing the vibrations of various types of structures, especially in tall buildings [3] or in bridges, for example, the Millennium Bridge [1] in London or bridges traversed by high-speed trains [2]. Moreover, absorbers can be used to prevent large vibration amplitudes that occasionally appear, that is, in geotechnical works [4].

The topic of dynamic vibration absorbers (DVAs) or tuned mass dampers (TMD) is well established in literature. The action of the absorber is accepted as a deterministic [5-9] or random process [10-20]. A large number of studies have been conducted to optimise the design parameters, and hence maximise the performance of vibration absorbers. Although the design concept of a TMD was developed decades ago, its simplicity and effectiveness have made it one of the most popular passive vibration methods to suppress structural vibration [21].

In the earlier years of the 20th century, Frahm [22] introduced the use of a linear spring mass attachment to suppress the oscillations of harmonically excited primary structural systems in engineering applications. This early DVA was able to reduce the oscillations of primary structures with a single degree of freedom, but could only reduce vibration transmission in a specific narrow frequency band. A spring-supported mass invented by Frahm is a dynamic absorber without damping, known as a Frahm damper. Ormondroyd and Den Hartog [23] increased the effectiveness of the absorber to dissipate the energy of the primary structure to harmonic excitations by appending a viscous damper parallel to the linear spring. Later, a semi-empirical design procedure was established by Den Hartog [5]. The DVA proposed by Den Hartog is now known as the Voigt-type DVA dynamic vibration absorber, where a spring element and a viscous element are connected. This is considered as the standard model of the DVA. The main objective in the design of the standard type DVA is to enable the absorber to have optimum

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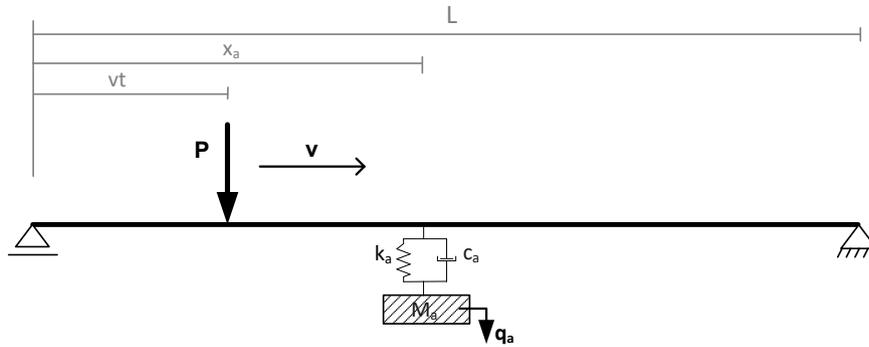
Figure 1: The model of the system.

parameters. Due to the fact that the mass ratio of the DVA to the primary structure is usually a few percent, the principal parameters of the DVA are its tuning ratio (i.e. the ratio of the DVA's frequency to the natural frequency of the primary structure) and damping ratio [24].

Ormondroyd and Den Hartog [23] developed analytical solutions for the optimal design of the classic DVA using the fixed-points technique. Den Hartog [5] provided an algorithm for selecting the optimal absorber parameters, so that the maximum amplitude of vibration is minimised over frequencies of deterministic sinusoidal excitations, which is essentially an H_1 optimisation. Asami and Nishihara used H_∞ and H_2 optimisation methods based on the perturbation technique and Vieta's theory to derive analytical solutions for the optimal design of the classic DVA [25-26]. Sims [27] introduced a new analytical solution based on the criterion of minimising either the positive real part or the negative real part of the frequency response function. Shen et al. [28] proposed a new strategy for obtaining the optimum negative stiffness ratio and to also make the system remain stable.

Most works assume that the load process is deterministic and that it changes harmonically. The problem of the optimisation of absorbers connected with a bridge beam loaded by a moving force was considered in papers [29-30]. The vibrations of some structures are excited by a load of random nature. There are relatively few studies on absorbers in which the load is a stationary stochastic process [31-34]. In the mentioned works, stochastic vibrations were analysed in the field of correlation theory and spectral density analysis.

The earliest research works on the optimal design of a DVA were presented for a single degree of freedom (SDOF) of a primary structure [5, 22, 23]. In the vast majority of papers, the structure-absorber system is still treated as a system with two degrees of freedom (2DOF). The optimal design of an absorber is not a new topic, and the

optimal methodologies and parameters of the optimal DVA or TMD system for SDOF main structures subjected to different loading conditions have been verified by several researchers since the 1960s [21].

The problem of choosing the optimal parameters for damping absorbers that reduce the random vibrations of a beam subjected to a sequence of moving forces with a constant velocity is studied in this paper. Every force is regarded as a random variable. Moreover, the inter-arrival times of moving forces are regarded as random variables. The stochastic properties of the load are modelled by means of a filtered Poisson process. The problem was solved with the idea of a dynamic influence function [35]. Several optimisation criteria, based on the expected values and variance of the beam response determined in the study, were considered.

2 Formulation of the problem and general solutions

Let us consider the damping vibrations of a simply supported Euler–Bernoulli beam of finite length L with an absorber fitted at point x_a subjected to a sequence of moving forces with a constant velocity v (see Figure 1). The vibrations of the beam are described by the equation [36]

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + c\frac{\partial w(x,t)}{\partial t} + m\frac{\partial^2 w(x,t)}{\partial t^2} + r(t)\delta(x-x_a) =$$
$$\sum_{k=1}^{N(t)} A_k\delta[x-v(t-t_k)] \tag{1}$$

where EI denotes the flexural stiffness of the beam, m denotes the mass per unit length of the beam, c is the

damping coefficient, $\delta(.)$ denotes the Dirac delta function and $r(t)$ is the force response of the absorber acting on the beam. The reaction of absorber on beam can be derived from equilibrium of the absorber.

The amplitudes A_k are assumed to be random variables, which are both mutually independent and independent of the random instants t_k. It is assumed that the expected values $E[A_k]=E[A]=$const are known. Random times t_k constitute a Poisson process $N(t)$ with a parameter λ.

For a finite, simply supported beam, the boundary conditions have the following forms:

$$w(0,t) = w(L,t) = 0, \tag{2}$$

$$\left.\frac{\partial^2 w(x,t)}{\partial x^2}\right|_{x=0} = \left.\frac{\partial^2 w(x,t)}{\partial x^2}\right|_{x=L} = 0. \tag{3}$$

Let us introduce two dynamic influence functions – $H_1(x,t)$ and $H_2(x,t-L/v)$. The function $H_1(x,t)$ is the response of the beam at time t $(0 \leq t \leq L/v)$ to a moving force equal to unity $(A_k=1)$, and the function $H_2(x,t-L/v)$ is the response of the system without excitation, but with non-zero initial conditions (the force has already left the beam). These influence functions satisfy the following differential equations:

$$EI\frac{\partial^4 H_1(x,t)}{\partial x^4} + c\frac{\partial H_1(x,t)}{\partial t} + m\frac{\partial^2 H_1(x,t)}{\partial t^2} + \\ r(t)\delta(x - x_a) = \delta(x - vt), \tag{4}$$

$$EI\frac{\partial^4 H_2\left(x,t-\frac{L}{v}\right)}{\partial x^4} + c\frac{\partial H_2\left(x,t-\frac{L}{v}\right)}{\partial t} + m\frac{\partial^2 H_2\left(x,t-\frac{L}{v}\right)}{\partial t^2} + \\ + r\left(t - \frac{L}{v}\right)\delta(x - x_a) = 0, \tag{5}$$

and appropriate boundary conditions and initial conditions

$$H_1(x,0) = \left.\frac{\partial H_1(x,t)}{\partial t}\right|_{t=0} = 0, \tag{6}$$

$$H_2(x,0) = H_1\left(x,\frac{L}{v}\right), \left.\frac{\partial H_2(x,t)}{\partial t}\right|_{t=0} = \left.\frac{\partial H_1(x,t)}{\partial t}\right|_{t=\frac{L}{v}}. \tag{7}$$

The deflection of the beam $w_I(x,t)$ is counted at arbitrary time t, and none of the force locations are known. It can be written in the Stieltjes integral:

$$w_I(x,t) = \int_{t-\frac{L}{v}}^{t} A(\tau)H_1(x,t-\tau)dN(\tau) +$$

$$+ \int_{t_b}^{t-\frac{L}{v}} A(\tau)H_2\left(x,t-\tau-\frac{L}{v}\right)dN(\tau). \tag{8}$$

The expected value of the random function $w_I(x,t)$ amounts to [37]

$$E[w_I(x,t)] = E[A]\lambda \int_{t-\frac{L}{v}}^{t} H_1(x,t-\tau)d\tau + \\ E[A]\lambda \int_{t_b}^{t-\frac{L}{v}} H_2\left(x,t-\tau-\frac{L}{v}\right)d\tau, \tag{9}$$

whereas the variance amounts to

$$\sigma_{wI}^2(x,t) = E[A^2]\lambda \int_{t-\frac{L}{v}}^{t} H_1^2(x,t-\tau)d\tau + \\ E[A^2]\lambda \int_{t_b}^{t-\frac{L}{v}} H_2^2\left(x,t-\tau-\frac{L}{v}\right)d\tau. \tag{10}$$

The symbol $E[.]$ denotes the expected value of the quantity in the brackets, v_A is the coefficient of variation and $E[A^2]=E^2[A](1+v_A^2)$. The parameter $\lambda>0$ is the intensity of the Poisson process and is equal to the average number of points per unit of time. The above general solution will be used to optimise the absorber parameters. For transient vibration, we assume $t_b=0$ and for steady state, $t_b=-\infty$. In the last case, equations (9) and (10) have the forms

$$E[w_I(x,\infty)] = E[A]\lambda \int_{0}^{\frac{L}{v}} H_1(x,\tau)d\tau + \\ + E[A]\lambda \int_{\frac{L}{v}}^{\infty} H_2\left(x,t-\tau-\frac{L}{v}\right)d\tau = \\ = E[A]\lambda E[w], \tag{11}$$

$$\sigma_{wI}^2(x.\infty) = E[A^2]\lambda \int_{0}^{\frac{L}{v}} H_1^2(x,\tau)d\tau + \\ + E[A^2]\lambda \int_{\frac{L}{v}}^{\infty} H_2^2\left(x,t-\tau-\frac{L}{v}\right)d\tau = \\ = E[A^2]\lambda \sigma^2. \tag{12}$$

3 Determination of the dynamic influence function

In the case of a simply supported beam, one can look for dynamic influence functions in the form of sine series

$$H_1(x,t) = \sum_{n=1}^{\infty} y_{1n}(t)\sin\frac{n\pi x}{L}, \qquad (13)$$

$$H_2\left(x, t - \frac{L}{v}\right) = \sum_{n=1}^{\infty} y_{2n}\left(t - \frac{L}{v}\right)\sin\frac{n\pi x}{L}. \qquad (14)$$

By substituting expressions (13) and (14) into equations (4) and (5), and using the orthogonality method (multiplying by $sin\frac{n\pi x}{L}$, integrating over $0 \leq x \leq L$ and taking into consideration j first eigenforms), one can obtain the following sets of ordinary equations:

$$\frac{d^2 y_{1n}(t)}{dt^2} + 2\alpha\frac{dy_{1n}(t)}{dt} + \omega_n^2 y_{1n}(t) +$$
$$+ r(t)\sin\frac{n\pi x_a}{L} = \frac{2}{mL}\sin\frac{n\pi vt}{L}, \qquad (15)$$

$$\frac{d^2 y_{2n}(t)}{dt^2} + 2\alpha\frac{dy_{2n}(t)}{dt} + \omega_n^2 y_{2n}(t) + r(t)\sin\frac{n\pi x_a}{L} = 0, \qquad (16)$$

where $= 1, 2, \ldots, j$, $2\alpha = \frac{c}{m}$, $\omega_n^2 = \left(\frac{n\pi}{L}\right)^4\frac{EI}{m}$. The parameter j in the tests varied from 1 to 20, while for further calculations, it was found that a sufficient number was $j = 5$. The initial conditions for the functions $y_{1n}(t)$ and $y_{2n}(t)$ result directly from conditions (6) and (7).

4 Absorber equations

The equations of motion and the $r(t)$ reaction of their impact on the beam will be derived. The vibrations of the SDOF absorber attached to the beam at point x_a are described by the following equation:

$$M_a\frac{d^2 q_a(t)}{dt^2} + c_a\left[\frac{dq_a(t)}{dt} - \frac{dH_i(x_a,t)}{dt}\right] +$$
$$+ k_a[q_a(t) - H_i(x_a,t)] = 0, \qquad (17)$$

which, after taking into account (13) or (14) and the j first eigenforms, has the form

$$\frac{d^2 q_a(t)}{dt^2} + 2\alpha_a\left[\frac{dq_a(t)}{dt} - \sum_{n=1}^{j}\frac{dy_{in}(t)}{dt}\sin\frac{n\pi x_a}{L}\right] +$$
$$+ \omega_a^2\left[q_a(t) - \sum_{n=1}^{j} y_{in}(t)\sin\frac{n\pi x_a}{L}\right] = 0, \qquad (18)$$

where $i=1, 2$ and $2\alpha_a = \frac{c_a}{M_a}$, $\omega_a^2 = \frac{k_a}{M_a}$.

The reaction $r(t)$ between the beam and the absorber is given by the following equation:

$$r(t) = M_a\frac{d^2 q_a(t)}{dt^2}. \qquad (19)$$

It is assumed that the reaction $r(t)$ acts upwards on the beam. After substituting dependence (19) to equations (15) and (16), two separate systems of ordinary differential equations, together with equation (18), are obtained, for which solutions are achieved by numerical integration using Wolfram Mathematica [38]. As a standard, this software uses the Runge–Kutta method, automatically selecting the integration step; in the calculations, the maximum integration step was limited to one-thousandth of the passing time.

5 Optimisation criteria and reliability assessments

The issue of optimising absorber parameters has been considered in many studies. The problem cannot always be solved in an analytical way, especially in the case of stochastic loads. Moreover, it is possible to adopt various optimisation criteria. The solution of the optimisation problem depends both on the speed of the moving force and the parameters of the beam.

Let us assume the following optimisation criteria:

$$\text{Min } E[w_I(x,\infty)] = \text{Min } E[w], \qquad (20)$$

$$\text{Min } \sigma_{wI}^2(x,\infty) = \text{Min } \sigma^2. \qquad (21)$$

Let h denote the maximum displacement of the beam in section x (e.g. $x = 0.5L$), which is caused by the passage of a unit force ($A = 1$):

$$h = H_1(x, t_i) = \text{Max } H_1(x,t). \qquad (22)$$

The knowledge of the quantity h allows the problem of parameter optimisation in terms of assessing the beam's reliability to be formulated. Let us assume that $F_A(x)$ denotes the log-normal distribution function of the random amplitude A_i, of which the expected value and variance are known. A force of A_i amplitude causes

maximum displacement of the beam equal to A_ih. Let function $F(x,t)_{max}$ determine the distribution of beam response maxima in a given period $(0,t)$ It is assumed that the time of a single passage is small enough, when compared to the time of exploiting the structure, that it can be omitted, and therefore, a single passage can be treated as an impulse. This function is given by the equation [39]:

$$F_{max}(x,t) = \sum_{i=1}^{\infty} P\{N(t) =$$

$$= k\} \prod_{i=1}^{j} \left\{ P\left(A_i \le \frac{x}{h}\right) \right\} = \sum_{j=1}^{\infty} p_n(t) F_A^j\left(\frac{x}{h}\right), \quad (23)$$

where $p_n(t)$ is the probability of j pulses in time $(0,t)$.

Considering that the $N(t)$ process is a Poisson process with parameter $\lambda>0$, the above equation can be presented in the form [39]:

$$F_{max}(x,t) = exp\left\{-\lambda t\left[1 - F_A\left(\frac{x}{h}\right)\right]\right\}. \quad (24)$$

The above solutions can be used to optimise the parameters of the absorber with respect to reliability. As the first criterion, we can assume a minimum probability of exceeding the permissible level Δ by a single pulse. This probability p_f is given by the equation [39]:

$$p_f = 1 - p_r = 1 - \Phi\left[\frac{ln\left(\frac{\Delta}{h}\right) - E[lnA]}{\sigma_{lnA}^2}\right], \quad (25)$$

where $\Phi(.)$ is the standard normal distribution.

The next criterion assumes the probability that the maximum vibrations in the period $(0,T)$ will not exceed Δ level. This probability is equal to [39]:

$$p_r(T) = 1 - p_f(T) = exp\left\{-\lambda T\left[1 - F_A\left(\frac{\Delta}{h}\right)\right]\right\}, \quad (26)$$

where

$$F_A\left(\frac{\Delta}{h}\right) = \Phi\left[\frac{ln\left(\frac{\Delta}{h}\right) - E[lnA]}{\sigma_{lnA}^2}\right] \quad (27)$$

and $\sigma_{lnA}^2 = ln(1+v_A^2)$, $E[lnA] = m_{lnA} = ln(E[A]) - \frac{1}{2}\sigma_A^2$.
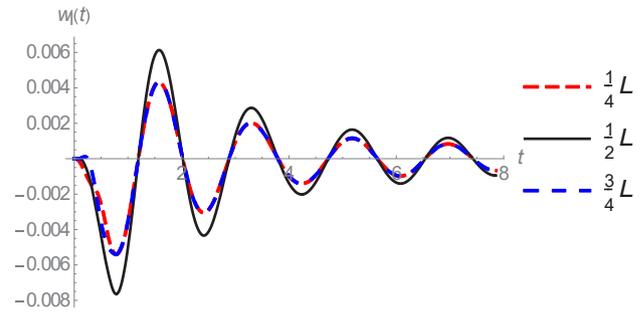
Figure 2: Deflection of the structure. An absorber with parameters μ=0.05, κ=1.

6 Numerical analysis

The model of the structure refers to a road bridge with a suspended platform equipped with municipal technical infrastructure. It is a reinforced concrete beam bridge with a span of L=30 m, in which the first eigenfrequency ω_s=4 rad/s. After analysing a range of 3–20 forms, it was found that 5 eigenforms can be adopted for further calculations. Different positions of the absorbers, $x_a \in (0,L)$, were considered. Various parameters of the absorber were analysed ($\mu = M_a/mL$, $\kappa = \omega_a/\omega_s$). Since $E[A]$, $E[A^2]$ and λ are constant for the homogeneous Poisson process, the figures below show the deflections, expected values and variances of displacements in the middle of the span of the bridge beam without the value of these parameters.

Figures 2 and 3 show that the deflection of the structure is in three places (x_a=L/4,L/2,3/4L). Figure 2 shows deflections of the structure with the absorber ($\mu = M_a/mL$=0.05, $\kappa = \omega_a/\omega_s$=1). It was assumed that the vehicle traffic moves at a critical speed, which was adopted according to the well-known formula ($v_{cr} = \pi\sqrt{EI/mL^2}$).

As shown in the figure above, the beam deflection in one-fourth of its length on the left and right sides is almost identical after the travel time has elapsed. Slight changes are visible at the beginning of the observation. A similar situation can be seen for the structure without an absorber (see Figure 3). It is obvious that deflection in the middle is greater than at one-fourth of the beam's length.

Various tuned absorbers were analysed. The influence of $\kappa = \omega_a/\omega_s$ parameter is definitely greater than that of mass $\mu = M_a/mL$. Figure 4 shows three deflections in the middle of the beam: the blue line is for a structure without an absorber, the orange line is for an absorber with parameters μ=0.05, κ=1 and the green line is for μ=0.05, κ=2.

Figure 5 shows three deflections in the middle of the beam: the blue line is for a structure without an absorber,
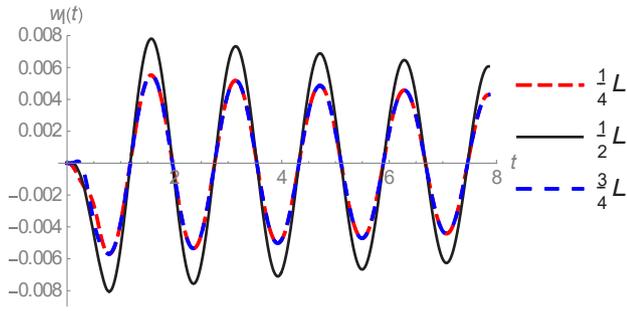
Figure 3: Deflection of the structure without an absorber.

Figure 4: Deflection of the structure in the midspan for two absorbers with different κ.

Figure 5: Deflection of the structure in the midspan for two absorbers with different μ.

Figure 6: Deflection of the structure without an absorber in the middle of the beam for three velocities.

Figure 7: Deflection of the structure with the absorber $\mu=0.05$, $\kappa=1$ in the middle of the beam for three velocities.

the orange line is for an absorber with parameters $\mu=0.05$, $\kappa=1$ and the green line is for $\mu=0.025$, $\kappa=1$.

The graphs below show the analysis of the absorber with the parameters $\mu=0.05$, $\kappa=1$. When comparing Figures 3–5, it can be seen that the absorber is well-tuned, which means that it reduces the vibrations of the beam. The influence of the velocity of mobile vehicles was also analysed (see Figures 6 and 7).

A wide range of analysis of the influence of the absorber mass was adopted for $\mu\in\langle0,0.3\rangle$. In practice, up to 10% is usually used (μ_{max}=0.1). As shown in Figure 8a, the expected value of the displacement in the middle of the span of the bridge beam, along with the equipment and technical infrastructure, does not depend on the absorber's mass. Similarly, a wide range of analysis of the influence of the absorber frequency was adopted,

$\kappa\in\langle0.5,2.5\rangle$. In practice, an absorber with a frequency similar to the structure's frequency ($\kappa\approx1$) is most often used. As shown in Figure 8b, the expected value of the displacement in the midspan does not depend on the absorber's frequency. However, the impact of the velocity of cars is very large. The analysed range for the selected structure is 68–336 km/h. A higher speed corresponds to a smaller deflection (the greater the influence of the absorber).

As shown in Figure 9a and b, the variance of these displacements also depends on velocity, but in a different way than is expected. The biggest variance of deflection is when the velocity of the passage is within the range 0.6–0.8 v_{cr} with regards to the analysis of both the μ and κ parameters. However, there is a slight influence of the mass ratio (see Figure 9a). Good results can be seen for $\mu\in\langle5\%,15\%\rangle$. When $\mu<5\%$ and $\mu>20\%$ for $v=(0.6-0.8)v_{cr}$, the variance increases. The influence of the frequency ratio can also be seen (see Figure 9b). The smaller the κ, the smaller the deflection is in the range of critical velocity.
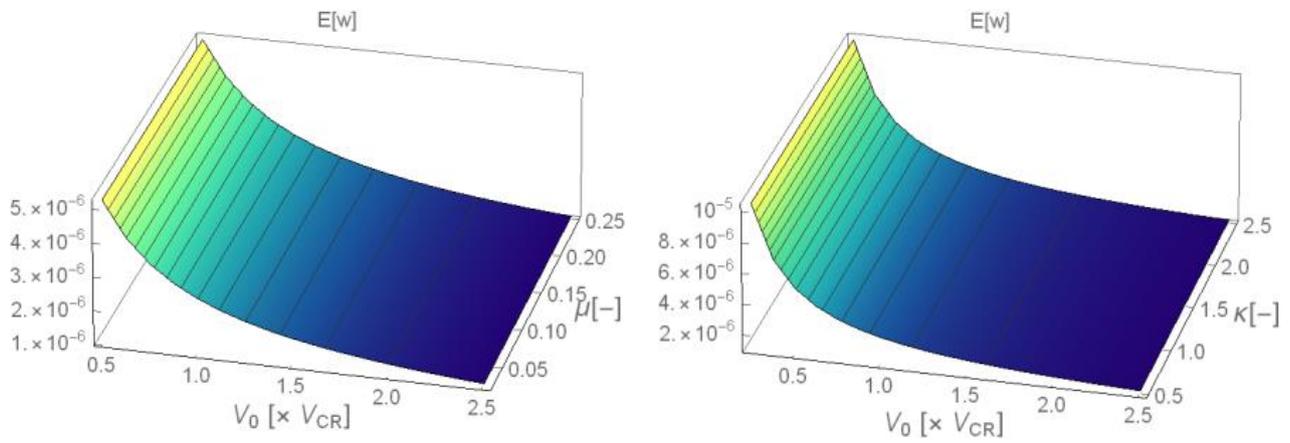
Figure 8: The expected value of deflections in the midspan for different vehicle velocities and also for **(a)** different parameters μ of the absorbers and **(b)** different parameters κ of the absorbers.
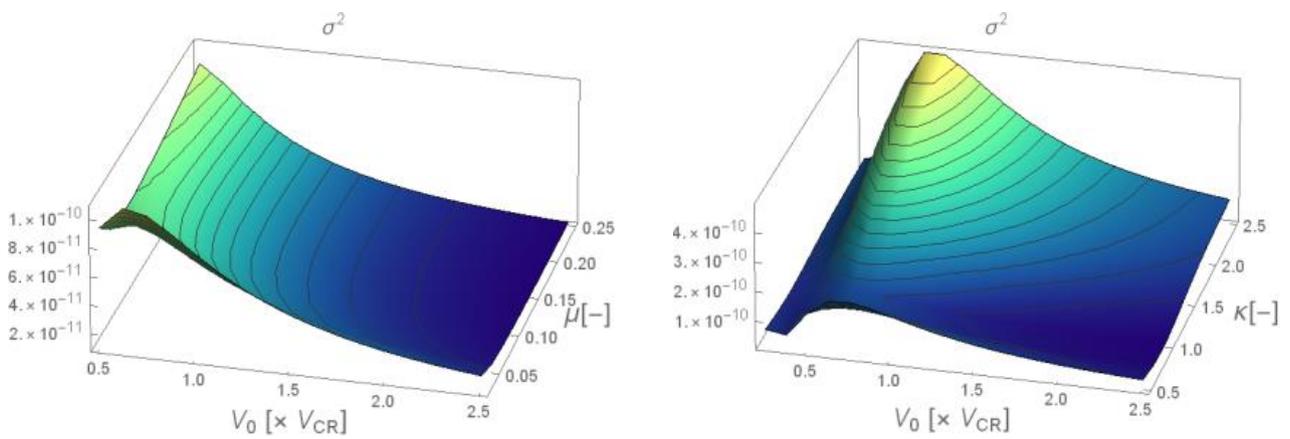
Figure 9: The variance of deflections in the midspan for different vehicle velocities, and also for **(a)** different parameters μ of the absorbers and **(b)** different parameters κ of the absorbers.
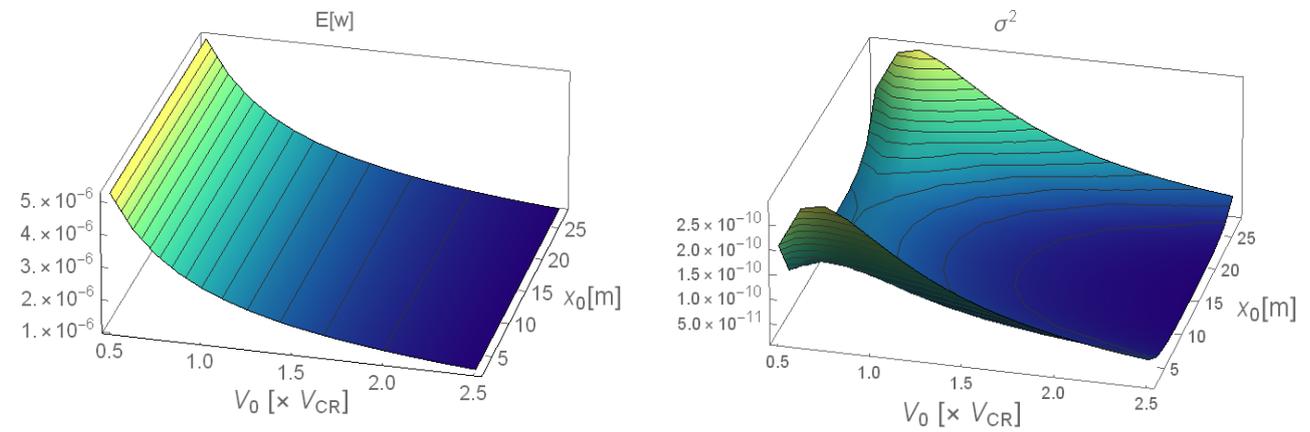
Figure 10: Deflections in the midspan for different locations of the absorber: **(a)** the expected value and **(b)** the variance.

The subject of the research was also the location of the absorber. Is it really the best solution to install the DVA in the middle of the beam? The authors obtained an answer that the optimal operation of the absorber is when it is installed in the range of 0.3–0.6 of the beam's span. Figure 10 shows the dependence between both the expected value and the variance of the midspan deflection and the absorber's position. The results are presented for different velocities of motor vehicles.

7 Conclusion and comments

According to Warburton [40], it is well-known that under the same optimisation procedure, the performance of the TMD system is better with an increase of the total mass of the attached TMD. This study analysed the range of the absorber mass from 0 to as much as 30% of a structure's mass. Years ago, it was accepted that the total mass of the attached absorbers should not exceed 10% of the mass of the main system.

After analysing the influence of the absorber's $\mu=M_a/mL$ and $\kappa=\omega_a/\omega_s$ parameters, as well as its location along the length of the bridge beam (together with technical infrastructure devices), it was found that the variance of displacements in the middle of the span should be taken into account when optimising the absorber. The smaller the variance, the better the tuned absorber is. It was found that the expected value of deflection is not significant when tuning the absorber.

In the case of the analysed reinforced concrete beam bridge with a span of 30 m, on which a platform, along with the technical infrastructure, was installed (e.g. hazardous gas), the optimal absorber has parameters in the following ranges: mass $M_a\leq0.05$ mL,0.10 mL> (5%–10% of the total mass of the structure) and $\omega_a=\omega_s$ (the absorber's frequency is equal to the structure's frequency.

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